فلسفه ریاضیات

۲۲-۱۲۶

نیمسال دوم ۱۴۰۳-۱۴۰۲

دانشکده علوم ریاضی، دانشگاه صنعتی شریف

A. Introductory Texts

- **Bostock, D.** *Philosophy of Mathematics: An Introduction* (2009)
- **Brown, J. R.** *Philosophy of Mathematics*, 2nded (2008)
- **Çevik, A.** *Philosophy of Mathematics: Classic and Contemporary* Studies (2022)
- **Colyvan, M.** *An Introduction to the Philosophy of Mathematics* (2011)
- **Friend, M.** *Introducing Philosophy of Mathematics* (2007)
- **George, A. and D. Velleman** *Philosophies of Mathematics* (2002)
- **Hamkins, J. D.** *Lectures on the Philosophy of Mathematics* (2020)
- **Körner, S.** *The Philosophy of Mathematics: An Introductory Essay* (1960)
- **Linnebo, Ø.** *Philosophy of Mathematics* (2017)
- **Ravn, O. and O. Skovsmose** *Connecting Humans to Equations* (2019)
- **Shapiro, S.** *Thinking about mathematics: The philosophy of mathematics* (2000)

- *Recommended as course texts*
- *Old but good*
- *Good, philosophy oriented*

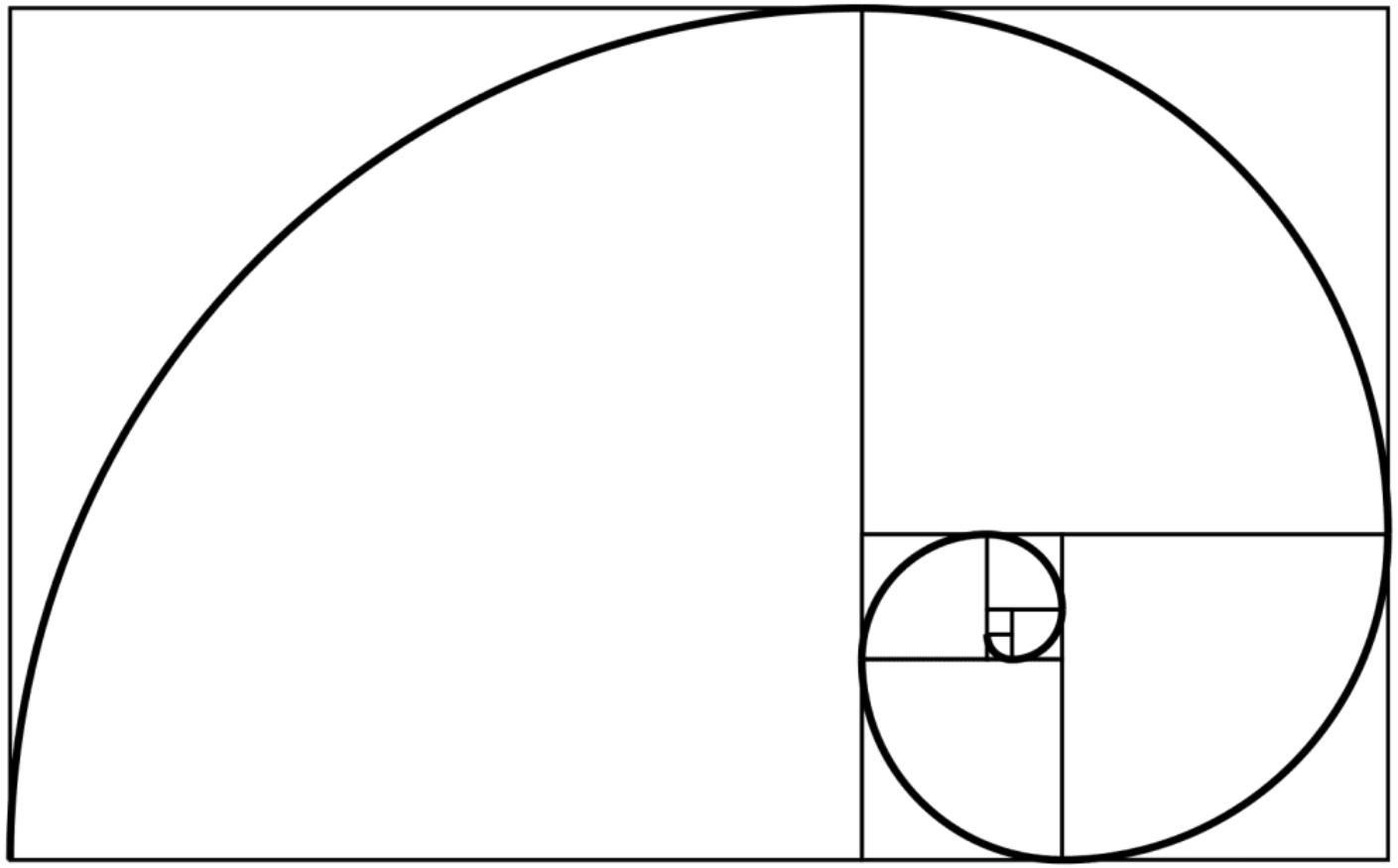
B. General Collections of Essays

- Benacerraf, P. and H. Putnam *Philosophy of Mathematics: Selected Readings*, 2nd ed (1983)
- Dybjer, P. et al *Epistemology versus Ontology* (2012)
- Ewald, W. *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, Vols. 1&2 (1996)
- Hart, W. *The Philosophy of Mathematics* (1996)
- Irvine, A.D. *Philosophy of Mathematics* (2009)
- Lindström, S., et al *Logicism, Intuitionism and Formalism: What has become of them?* (2009)
- Shapiro, S. *The Oxford Handbook of the Philosophy of Mathematics and Logic* (2005)
- Van Heijenoort, J. *From Frege to Gödel: A Source Book in Mathematical Logic* (1967)

Chronology: `Greek' Mathematics

Thales	-590	Euclid	-300
Pythagoras	-530	Archimedes	-250
Theaetetus	-390	Apollonius	-220
Plato	-380	Nicomachus	+90
Eudoxus	-370	Diophantus	+250
Aristotle	-350	Pappus	+300

مستطیل طلایی



مراجع اصلی افلاطون

- ***Collected Dialogues, including the Letters***

  Hamilton & H. Cairns (Eds.), Princeton U. Press 1963

- Wedberg, A. ***Plato's Philosophy of Mathematics***

  Almquist and Wiksell 1955

گفتگوهای اصلی مربوط به ریاضیات و معرفت شناسی

- **Meno, Republic**
- **Phaedo, Theaetetus, Parmenides, Philebus, Timaeus**
- **Gorgias, Phaedrus, Laws**

مراجع اصلی ارسطو

- ***The Basic Works of Aristotle***
  McKeon, R. (ed.), Modern Library 2001
- McKeon, R. ***Introduction to Aristotle***, Modern Library 1947
- Bostock, D. `Aristotle's Philosophy of Mathematics' in
  Oxford Handbook of Aristotle (2012)
- Mendell, H. `Aristotle and Mathematics', in
  Stanford Encyclopedia of Philosophy
  https://plato.stanford.edu/entries/aristotle-mathematics/

ارجاعات به آثار ارسطو

- ***Metaphysica*** (مابعدالطبیعه) **:** I.9, II.2, III.5-6, V.2, IX.9, X.1-3, XI.10-12, XIII.1-10, XIV.1-6
- ***Physica*** (طبیعیات) **:** II.2-3, III.4-8, V.3, VI.1-10
- ***Categoriae*** (مقولات) : V.6
- ***Analytica Priora*** (منطق) : I.1-13, 27-30
- ***Analytica Posteriora*** (آنالوطیقای‌دوم) **:** I.1-2, I.7, I.26
- ***De Anima*** (روح) **:** III.3, III.7
- ***De Caelo*** (سماوات) **:** I.2, I.5-7, II.13-14

ماهیت اشیاء ریاضی

- The faculty of thinking then thinks the forms in images…the mind when it is thinking the objects of Mathematics thinks as separate, elements that do not exist as separate. In every case the mind which is actively thinking is the objects which it thinks. ***De Anima*****:** III.7

- Now the mathematician, though he too treats these things, nevertheless does not treat them as limits of physical body; nor does he consider the attributes indicated as the attributes of such bodies. That is, he separates them; for in thought they are separable from motion, and it makes no difference, nor does any falsity result, if they are separated. ***Physica***: II.2

Analytica Posteriora, I.2 مبنای دانش علمی

We suppose ourselves to possess unqualified scientific knowledge of a thing, as opposed to knowing it in an accidental way… when we know the cause on which the fact depends…
What I now assert is that at all events we do know by demonstration. I mean a syllogism productive of scientific knowledge, a syllogism, that is, the grasp of which is *eo ipso* such knowledge. Assuming then my thesis as to the nature of scientific knowing is correct, the premises of demonstrated knowledge must be true, primary, immediate, better known than and prior to the conclusion, which is further related to them as effect to cause… The premises must be true: for that which is non-existent cannot be known – we cannot know, e.g., that the diagonal of a square is commensurable with side.

Analytica Posteriora, I.2

مبنای دانش علمی

We suppose ourselves to possess unqualified scientific knowledge of a thing, as opposed to knowing it in an accidental way… when we know the cause on which the fact depends…What I now assert is that at all events we do know by demonstration. I mean a syllogism productive of scientific knowledge, a syllogism, that is, the grasp of which is *eo ipso* such knowledge. Assuming then my thesis as to the nature of scientific knowing is correct, the premises of demonstrated knowledge must be true, primary, immediate, better known than and prior to the conclusion, which is further related to them as effect to cause…

Discrete vs Continuous: *Categoriae*, V.6

Quantity is either discrete or continuous…Instances of discrete are numbers and speech; of continuous, lines, surfaces, solids, and, besides these, time and place…
In the case of the parts of a number, there is no common boundary at which they join. For example: two fives make ten, but the two fives have no common boundary, but are separate; the parts three and seven also do not join at any boundary… Number, therefore, is a discrete quantity…A line, on the other hand, is a continuous quantity, for it is possible to find a common boundary at which its parts join. In the case of the line, this common boundary is the point…

Discrete vs Continuous: *Analytica Posteriora*, I.7

It follows that we cannot in demonstrating pass from one genus to another. We cannot, for instance, prove geometrical truths by arithmetic. For there are three elements in demonstration: (1) what is proved, the conclusion – an attribute inhering essentially in a genus; (2) the axioms, i.e., the axioms which are premises of demonstration; (3) the subject genus whose attributes, i.e., essential properties, are revealed by the demonstration. The axioms which are the premises of demonstration may be identical in two or more sciences: but in the case of two different genera such as arithmetic and geometry you cannot apply arithmetical demonstration to the properties of magnitudes unless the magnitudes in question are numbers…(cont.)

Discrete vs Continuous: *Analytica Posteriora*, I.7

Arithmetical demonstration and the other sciences likewise possess, each of them, their own genera; so that if the demonstration is to pass from one sphere to another, the genus must be either absolutely or to some extent the same. If this is not so, transference is clearly impossible, because the extreme and the middle terms must be drawn from the same genus: otherwise as predicated, they will not be essential and will thus be accidents. That is why it cannot be proved by geometry that …the product of two cubes is a cube.

Summary Contents of Euclid's *Elements*

Book	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	Totals
Definitions	23	2	11	7	18	4	22	–	–	16	28	–	–	131
Postulates	5	–	–	–	–	–	–	–	–	–	–	–	–	5
Common Notions	5	–	–	–	–	–	–	–	–	–	–	–	–	5
Propositions	48	14	37	16	25	33	39	27	36	115	39	18	18	465

https://en.wikipedia.org/wiki/Euclid%27s_Elements
(February 9,2024)

منابع ریاضیات دوره اسلامی

- منابع اولیه به عربی و ترجمه‌های فارسی و زبان‌های اروپایی
- مقالات متعدد به‌خصوص از Roshdi Rashed و Jan Hogendijk
- از دوکتاب زیر خیلی استفاده شده است:

Rashed, R. ***The Development of Arabic Mathematics: Between Arithmetic and Algeba***,1994

Rashed,R. & B. Vahabzadeh

Omar Khayyam, the Mathematician, 2000

مطالب این کتاب شامل محاسباتی است درارث ووصیت ومقاسمه (= تقسیم کردن اموال مشترك) و اموردیوانی وتجارت، و نیز درمورد تمام اموری که به‌حساب ومعامله مربوط می‌شود ـ مانند: مساحت کردن زمینها و اندازه گیری نهرها و هندسه (= نقشه کشی) و دیگر مباحث و فنون ریاضی ـ قابل استفاده خواهد بود. این کتاب را با حسن‌نیتی که

- از مقدمه *جبر و مقابله* خوارزمی، ترجمه حسین خدیوجم
- انتشارات اطلاعات، ۱۳۶۳

بحث ائودوکسوس (فصل پنج اقلیدس) از مقایسه دو نسبت :

A و **B** دو مقدار از یک کمیت پیوسته همگن، و **C** و **D** دو مقدار از یک کمیت پیوسته همگن (دیگر) هستند . نسبت **(A:B)** را برابر نسبت **(C:D)** می‌نامیم در صورتی که به‌ازای هر دو عدد صحیح(مثبت) **m** و **n** (حداکثر) دفعاتی که **nB** در **mA** می گنجد برابر (حداکثر) دفعاتی باشد که **nD** در **mC** می گنجد . چنانچه تعداد دفعات مربوط به **(A:B)** از تعداد دفعات مربوط به **(C:D)** کوچک‌تر باشد ، نسبت **(A:B)** ازنسبت **(C:D)** بزرگتر است.

توجه : برابری دو نسبت ناگویا در تعدادی متناهی گام نتیجه نمی‌شود .

بحث خیام از مقایسه دو نسبت :

A و **B** دو مقدار از یک کمیت پیوسته همگن، و **C** و **D** دو مقدار از یک کمیت پیوسته همگن (دیگر) هستند . برای نسبت **(A:B)** نمایش کسر مسلسل $[r_0;r_1,r_2,r_3,...]$ وبرای نسبت نمایش **(C:D)** کسر مسلسل $[s_0;s_1,s_2,s_3,...]$ را درنظر می گیریم . اگر به‌ازای هر **n** ، r_n و s_n برابر باشند می گوییم دو نسبت برابرند . فرض کنید **n** کوچکترین مرتبه‌ای باشد که مثلا $r_n < s_n$ چنانچه **n** فرد باشد، نسبت **(A:B)** از نسبت **(C:D)** بزرگتر است، و چنانچه **n** زوج باشد، نسبت **(A:B)** از نسبت **(C:D)** کوچکتر است

توجه : برابری دو نسبت ناگویا در تعدادی متناهی گام نتیجه نمی‌شود .

منابع فلسفه ریاضیات دوره اسلامی

Ardeshir, M. `Ibn Sina's Philosophy of Mathematics,' in ***The Unity of Science in the Arabic Tradition***, 2008

Zarepour,M.S. `Arabic and Islamic Philosophy of Mathematics,' in ***Stanford Encyclopedia of Philosophy***, 2002 https://plato.stanford.edu/entries/arabic-islamic-phil-math/

Wisnovsky, R. ***Aspects of Avicenna***, 2001 (Articles by D. Gutas and D.N. Hasse)

Black, D.L. `Estimation in Avicenna: The Logical and Psychological Dimensions,' ***Dialogue*** 32, 219-58 (1993)

Greek → Arabic → Latin

Thales	-590	Pappus	+320	Samaw'al	+1150
Pythagoras	-530	Eutocius	+510	Tusi, Nasireddin	+1200
Theaetetus	-390	Khwarizmi	+810	Fibonacci	+1202
Eudoxus	-370	Mahani	+820	Kashi	+1420
Euclid	-300	Karaji	+950	Cardano	+1545
Archimedes	-250	Ibn-Haytham	+1020	Viète	+1570
Apollonius	-220	Khayyam	+1090	Stevin	+1585
Diophantus	+250	Tusi, Sharafeddin	+1135	Descartes	+1637

رومی‌ها و ریاضیات

- نقل از **Cicero** :

یونانیان هندسه را بسیار ارج می‌نهادند و از اینرو برای هیچکس به اندازه ریاضی‌دانان احترام قائل نبودند، ولی رومیان این فن را به وسیله‌ای برای اندازه‌گیری و محاسبه محدود کرده‌اند .

نقل از **Plutarch** :

فن ساختن ابزار هندسی را آرخیتاس و ائودوکسوس ابداع کردند و موفق شدند بعضی ترسیمات را که با خطکش و پرگار ممکن نبود با این ابزار انجام دهند . ولی افلاطون آنها را به اتهام تنزل دادن مقام هندسه سرزنش کرد . . . به این سبب مکانیک از هندسه جدا شد و از جرگه فلسفه به یک فن نظامی مبدل گشت .

اروپای قرن شانزده

Gerolamo Cardano	***Ars Magna* (1545)** حل جبری معادلات درجه۳ و ۴، پذیرش اعداد منفی و به‌کارگیری محدود اعداد موهومی
Simon Stevin	***De Thiende* (1585), *L'arithmétique* (1585)** رواج دادن عددنویسی (کسری) اعشاری در اروپا، یکپارچه کردن مفهوم عدد حقیقی، قضیه مقدار بینی برای چندجمله‌ایها
François Viète	***Algebra Nova* (1591), *Supplementum geometriae* (1593)** آغاز استفاده کامل از جبر نمادین، پیشتاز هندسه تحلیلی

René Descartes (1596-1650)

- **Here I beg you to observe in passing that the scruples that ancient writers observed in using arithmetical terms in geometry, thus making it impossible for them to proceed beyond a point where they could see clearly the relation between the two subjects, caused much obscurity and embarrassment, in their attempts at explanation.**
- **I would borrow the best of geometry and of algebra and correct all the faults of the one by the other.**

اولین جملات کتاب هندسه دکارت:

Any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction. Just as arithmetic consists of only four or five operations, namely addition, subtraction, multiplication, division and the extraction of roots, which may be considered a kind of division, so in geometry, to find required lines it is merely necessary to add or subtract other lines; or else, taking one line which I shall call unity in order to relate it as closely as possible to numbers, and which can in general be chosen arbitrarily …

مراجع دكارت:

- ***The Geometry of Rene Descartes with a facsimile of the first edition*** (Dover 1954)
- ***Discourse on the Method, Optics, Geometry and Meteorology*** (Hackett 2001)
- ***Meditations on First Philosophy*** (Cambridge 1996)

اخترشناسی و فیزیک قرون ۱۶ و ۱۷ قبل از نیوتن

Nicolaus Copernicus (1473-1543)
Galileo Galilei (1564-1642)
Johannes Kepler (1571-1630)

قوانین کپلر:

۱ – سیارات در مدارهای بیضی دور خورشید می‌گردند و خورشید در یک کانون بیضی قرار دارد.

۲ – در زمان‌های مساوی شعاع حامل از سیاره به خورشید مساحت مساوی می‌پیماید.

۳ – مجذور زمان یک دور مدار متناسب با مکعب نیم شعاع بزرگ بیضی است.

حسابان قرون ۱۶ و ۱۷ قبل از نیوتن و لایب‌نیتس

- **Johannes Kepler** (1571-1630)
- **Bonaventura Cavalieri** (1598-1647)
- **Pierre de Fermat** (1601-1665)
- **John Wallis** (1616-1703)
- **Blaise Pascal** (1623-1662)
- **Isaac Barrow** (1630-1677)

نقل از مقدمه چاپ نشده کتاب *پرینکیپای* نیوتن:

The ancient geometers investigated by analysis what was sought, demonstrated by synthesis what had been found, and published what had been demonstrated so that it might be received into geometry. What was resolved was not immediately received into geometry; a solution by means of the composition of demonstrations was required. For all the power and glory of geometry consisted in certainty of things, and certainty consisted in demonstrations clearly composed. In this science what counts is not so much brevity as certainty. And accordingly, in the following treatise I have demonstrated by synthesis the propositions found by analysis.

نقل از توضیحات یازده لم فصل اول کتاب نیوتن (۱) :

... این لم‌ها را قبل از گزاره‌ها از اینرو آورده‌ام که از برهان‌های طولانی و کسل‌کننده به روش برهان خلف که شیوه هندسه‌دانان باستانی است احتراز کنم. اثبات با توسل به تجزیه‌ناپذیرها [indivisibles = بینهایت کوچکها] درواقع کوتاهتر است ولی از آنجا که تجزیه‌ناپذیرها مساله‌دار هستند و کمتر هندسی فرض می‌شوند، ترجیح داده‌ام برهان‌های بعدی را بر اساس مجموع‌ها و نسبت‌های غائی کمیت‌های میرا استوار کنم، یعنی حدّ چنین مجموع‌ها و نسبت‌ها....

نقل از توضیحات یازده لم فصل اول کتاب نیوتن (۲) :

...ممکن است ایراد گرفته شود که چیزی به عنوان نسبت غائی کمیت‌های میرا وجود ندارد زیرا قبل از صفر شدن نسبت غائی نیست، و پس از آن دیگر وجود ندارد، ولی با همین استدلال سرعت یک جسم در یک نقطه هم معنی ندارد زیرا قبل از آنکه به آن نقطه برسد سرعت غائی نیست و وقتی به آن نقطه برسد سرعتی در کار نیست ...

نسبت‌های غائی کمیت‌های میرا در واقع نسبت‌های کمیت‌های غائی نیستند، بلکه حدّ نسبت‌هایی هستند که این کمیت‌ها می‌گیرند ...

این موضوع را بهتر می‌توان درمورد کمیت‌هایی که بی‌انتها بزرگ می‌شوند دریافت. اگر دو کمیت هردو به بینهایت میل کنند، نسبت آنها می‌تواند به مقدار معینی میل کند ...

G.W.Leibniz (1646-1716)

ابداع نمادهای متداول و کارساز:

dx, dz/dx=(dz/dy).(dy/dx), $\int f(x)dx$

توسعه ریاضیات و فیزیک ریاضی در اروپای قاره‌ای با استفاده از

حسابان لایب‌نیتس در قرون ۱۷ و ۱۸ توسط:

L'Hospital, Bernoulli family, Euler, Lagrange, d'Alembert, Laplace, Legendre, Fourier, …

مراجعی برای لایب‌نیتس

- Goethe, Beeley and Rabouin (eds.):
  G. W. Leibniz, Interrelations between
  Mathematics and Philosophy
- Russell, B. *A Critical Exposition of the Philosophy of Leibniz*

نقل از نامه‌ای به مخاطبی در هلند (۱۶۹۴):

Whether infinite extensions successively greater and greater, or infinitely small ones successively less and less, are legitimate considerations, is a matter that I own to be possibly open to question;… it will be sufficient …that the error that anyone may assign may be less than a certain assigned quantity … it will be sufficient simply to make use of them as a tool that has advantages for the purpose of calculation, just as the algebraists retain imaginary roots with great profit.

تجربه گرایان بریتانیائی

- **John Locke** (1632-1704) *An Essay on Human Understanding* (1689)

  جستاری در خصوص فاهمه بشری (ترجمه کاوه لاجوردی)

- **George Berkeley** (1685-1753)

  A Treatise Concerning the Principles of Human Knowledge (1710)

  The Analyst: A Discourse Addressed to an Infidel Mathematician (1734)

- **David Hume** (1711-1776)

  A Treatise of Human Nature (1739-40)

  An Enquiry Concerning Human Understanding (1748)

  کاوشی در خصوص فهم بشری (ترجمه کاوه لاجوردی)

نقل از *آنالیزدان* بارکلی:

And what are these fluxions? The velocities of evanescent increments? And what are these same evanescent increments? They are neither finite quantities, nor quantities infinitely small, nor yet nothing. May we not call them ghosts of departed quantities?

و این *فلوکسیون‌ها* چه هستند؟ سرعت نموهای میرا (ناپدیدشونده)؟ و خود این نموهای میرا چه هستند؟ اینها نه کمیت‌های متناهی هستند، نه کمیت‌های بینهایت کوچک، و نه اینکه هیچ هستند. شاید بتوانیم آنهارا ارواح کمیت‌های محوشونده بنامیم؟

Immanuel Kant (1727-1804)

زاده و زیسته در Königsberg پروس (اکنونKaliningrad در روسیه)

- ***Critique of Pure Reason*** (1781, 1787)

  سنجش خرد ناب (میرشمس‌الدین ادیب سلطانی)، *نقد عقل محض* (بهروز نظری)

  B 1-30, B 31-73, B 176-187, B 204-205, B 741-766

- ***Prolegomena to Any Future Metaphysics*** (1783)

  تمهیدات (غلامعلی حداد عادل)

  4:255-305, Sections 1-22

- ***Prize Essay*** (1764)
- ***Inaugural Dissertation*** (1770)

منابع ثانوی مورد استفاده

- Posy, C. (Ed.) ***Kant's Philosophy of Mathematics: Modern Essays***, 1992. (Especially articles by J. Hintikka and C. Parsons.)
- Journal articles by **Michael Friedman** and **Emily Carson**
- Janiak, A. `Kant's View on Space and Time,' in

  Stanford Encyclopedia of Philosophy

  https://plato.stanford.edu/entries/kant-spacetime/#WhatTranReal (2022)
- Shabel, L. `Kant's Philosophy of Mathematics,' in

  Stanford Encyclopedia of Philosophy

  https://plato.stanford.edu/entries/kant-mathematical (2013)

چند کتاب جدیدتر:

- Carson, E. and L. Shabel ***Kant: Studies on Mathematics in the Critical Philosophy***, 2017
- Posy, C. and O. Rechter ***Kant's Philosophy of Mathematics, Volume1: The Critical Philosophy and its Roots***, 2020
- Shabel, L. ***Mathematics in Kant's Philosphy: Reflections on Practice***, 2003
- Sutherland, D. ***Kant's Mathematical World: Mathematics, Cognition and Experience***, 2022

واژگان کانت

آلمانی	انگلیسی	فارسی
Anschauung	Intuition	شهود
Begriff	Concept	مفهوم
Einzelnes	Singular	منفرد، تکین
Empfindung	Sensation	احساس
Erkenntis	Cognition	شناخت
Gemüth	Mind	ذهن
Sinnlichkeit	Sensibility	حساسیت، توان احساس
Urteil	Judgment	داوری، حکم
Verstand	Understanding	فهم، فاهمه
Vorstellung	Representation	نمایش

قطعاتی از تمهیدات، بخش ۱۳

- زمانی ریاضی‌دانانی که فیلسوف نیز بودند به شک افتادند ...در مورد اعتبارعینی ... هندسه زیرا نگران بودند که مبادا خط در طبیعت ممکن است واقعا از نقاط فیزیکی تشکیل شده باشد ...علی‌رغم اینکه فضایی که هندسه‌دان در ذهن دارد [چنین نیست] ...آنها متوجه نبودند که [این فضای هندسی] فضای اشیاء واقعی نیست، بلکه صورتی از نمایش حسی ماست، همه اشیاء آن فقط ظاهر هستند، نه خود اشیاء، بلکه نمایش آنها در شهود حسی ما ... (از *یادداشت ۱*، *A 287*)

قطعاتی از تمهیدات، بخش ۱۳

- ایده‌آلیسم این ادعاست که چیزی جز موجودات تفکر وجود ندارد و هر آنچه تصور می‌کنیم نسبت به آن ادراک داریم فقط نمایشی درذهن موجودات متفکرند و چیزی در خارج متناظر با آن وجود ندارد. من در مقابل می‌گویم: چیزهایی در بیرون وجود دارند که اشیاء مورد ادراک ما هستند، در عین حال چیزی از آنها به گونه‌ای که خود ممکن است باشند نمی‌دانیم، بلکه آشنایی ما فقط از طریق ظاهر است، یعنی نمایشی که برحواس ما می‌نهند ... (*از یادداشت ۲، A289*)

قطعاتی از تمهیدات، بخش ۱۳

- در گذشته تصور می‌شد که حسیات ما تصویر درهمی از واقعیات بیرونی به‌دست می‌دهند که نمی‌توان به‌گونه‌ای روشن در آگاهی ما نمایش داد، ما برعکس نشان دادیم که [مساله] حسیات تفاوت منطقی میان روشنی و ابهام نیست، بلکه تفاوت ریشه‌ای در منشاء خود شناخت است، زیرا که شناخت حسی چیزهارا اصلا آن‌طوری که هستند نمایش نمی‌دهند، بلکه به‌گونه‌ای که بر خواس ما اثر می‌گذارند [نمایش می‌دهند] (از *یادداشت۳* ،*A 291*)

Quotations from Eric Kandel:

- **… the belief that our perceptions are precise and direct is an illusion – a perceptual illusion. The brain does not simply take the raw data that it receives through the senses and reproduces it …each sensory system first analyzes and deconstructs, then reconstructs … incoming information according to its own built-in connections and rules – shades of Immanuel Kant!** - *In Search of Memory,* p.302
- ***Aplysia*'s neural circuit proved surprisingly invariant. Not only does every animal use the same cells in the reflex circuit, but also those cells are interconnected in precisely the same way in every animal. Each sensory cell and each interneuron connects connects to a particular set of target nerves … the first insight into Kantian *a priori*...**

- *Reductionism in Art and Brain Science*, p.51

From the *Critique*: A714/B742 (1)

Philosophical cognition is rational cognition from concepts, mathematical cognition that from the construction of concepts. But to construct a concept means to exhibit a priori the intuition corresponding to it. For the construction of a concept, therefore, a nonempirical intuition is required, which consequently, as intuition, is an individual object, a but that must nevertheless, as the construction of a concept (of a general representation), express in the representation universal validity for all possible intuitions that belong under the same concept.(cont.)

From the *Critique*: A714/B742 (2)

Thus, I construct a triangle by exhibiting an object corresponding to this concept, either through mere imagination, in pure intuition, or on paper, in empirical intuition, but in both cases completely a priori, without having had to borrow the pattern for it from any experience. The individual drawn figure is empirical, and nevertheless serves to express the concept without damage to its universality …

From the *Critique*: A714/B742 (3)

Philosophical cognition thus considers the particular only in the general; mathematical cognition considers the general in the particular, nay, even in the particular instance, but nonetheless does so *a priori* and by means of reason, in such a way that, just as this single instance is determined under certain universal conditions of construction, so too the object of the concept … must be thought as universally determined.

How is Pure Mathematics Possible?

How now is a great body of cognition…which carries apodictic certainty …hence rests on no grounds of experience, and so is a pure product of reason, but beyond this is thoroughly synthetic. “How is it possible then for human reason to achieve such a cognition wholly *a priori*?”

….all mathematical cognition … must present its concept beforehand *in intuition* and indeed *a priori* … in intuition that is not empirical but pure …

- Prolegomena, Section 6

Bernard Bolzano (1781-1848)

- **Considerations on Some Objects of Elementary Geometry** (1804)
- **Contributions to a Better-Grounded Presentation of Mathematics** (1810)
- **Purely analytic proof of the theorem that between any two values which give results of opposite sign, there lies at least one real root of the equation** (1817)
- **Paradoxes of the Infinite** (1850)

Johannes Herbart (1776-1841)

امروز به عنوان یکی از پایه‌گذاران روانشناسی و علم تعلیم و تربیت مطرح است، ولی زمان خود فیلسوف محسوب می‌شد. نفوذ او بر دانشمندان زیر در روش علمی، به خصوص رویکرد مفهومی است

- **Bernhard Riemann** (1826-1866)
- **Hermann Grassmann** (1809-1877)
- **Ernst Mach** (1838-1916)

پیرامون فرض‌هایی که هندسه برمبنای آن قرار دارد

چنان که همه می‌دانند موضوع هندسه، مفهوم فضا و اصول اولیه ساخت در فضا است. این چیزها فقط به صورت اسمی تعریف می‌شوند ولی خصوصیات اساسی‌شان توسط اصول موضوع مشخص می‌شود. روابط میان این پیش‌فرضها به شکل مبهم رها می‌شود و معلوم نمی‌گردد که آیا ارتباط میان آنها ضروری است، و اگر هست به چه میزان، آیا این ارتباط پیشینی است، و آیا حتی چنین ارتباطی امکان پذیراست؟ از اقلیدس تا لژاندر (که از معروفترین اصلاح‌گران نام برده باشم) این ابهام را نه ریاضی‌دانان برطرف کرده‌اند و نه فلاسفه. بی‌شک دلیل این است که مفهوم کلی کمیت چند بعدی (که کمیات هندسی را نیز شامل می‌شود) تاکنون مورد بررسی قرار نگرفته است.

چند نکته خطابه ریمان

- مطرح کردن زیرساخت کمیت چند بعدی (خمینه) به عنوان جایگاه هندسه
- اینکه بینهایت نوع هندسه می‌توان روی یک خمینه وضع کرد
- هندسه محدودیت بُعد ندارد، حتی هندسه‌های بینهایت بعدی و صفربعدی قابل طرح‌اند
- تفکیک هندسه و کیهان‌شناسی در رابطه با فیزیک جهان موجود

Richard Dedekind (1831-1915)

- Dirichlet's *Vorlesungen über Zahlentheorie* (1863)
- *Stetigkeit und irrationale Zahlen* (1867)
- *Was sind und was sollen die Zahlen* (1872)

ترجمه انگلیسی دو اثر آخر:

Essays on the Theory of Numbers (1963)

ابداع کننده اصطلاحات

Ideal, field, module, algebraic integer, isomorphism, homomorphism, system=set, ∋

From the Preface to: ***Was sind und was sollen die Zahlen***

از گفتن اینکه حساب (جبر، آنالیز) جزئی از منطق است مقصودم این است که مفهوم عدد را کاملا مستقل از ایده‌های شهودی فضا و زمان می‌دانم، و آن را نتیجه مستقیم قوانین تفکر به‌شمار می‌آورم ....

اعداد آفریده‌های آزاد ذهن انسانند؛ آنها وسیله‌ای برای درک آسان‌تر و دقیق‌تر تمایز اشیاء هستند. تنها توسط فرایند صرفا منطقیِ بناکردن علم اعداد و بدین ترتیب دست‌یافتن به قلمرو پیوسته اعداد است که امکان بررسی دقیق فهم ما از فضا و زمان، با ایجاد ارتباط آنها با قلمرو عددی ذهن، فراهم می‌گردد.

تعریف ددکیند از عدد طبیعی، ص.۶۸

73. **Definition**. If in the consideration of a simply infinite system N set in order by a transformation ϕ we entirely neglect the special character of the elements; simply retaining their indistinguishability and taking into account only the relations to one another in which they are placed by the order-setting transformation ϕ, then are these elements called *natural numbers* … and the base-element 1 is called the *base-number* of the *number series* N. With reference to this freeing the elements from every other content (abstraction) we are justified in calling numbers a free creation of the human mind.

Georg Cantor (1845-1918)

- نظریه مجموعه‌ها
- انواع بینهایت
- ترامتناهی‌های کاردینال و اردینال
- فرض پیوستار
- توپولوژی مجموعه نقاط
- پارادکس‌ها

Axiomatic Set Theory

- Ernst Zermelo, 1908
- Zermelo-Fraenkel, 1921, contributions by Skolem and von Neumann
- Formulated in first order logic. Undefined: Set, ∈
- ZFC = ZF + Axiom of Choice or equivalent

- Axiom of Extensionality: Sets are determined by their extensions
- Axiom of Regularity: Every non-empty set S has an element x so that S and x have empty intersection. (Together with Axiom of Pairing and extensionality bans paradoxes involving x ∈ x ; let S = {x}.)
- Axiom of Infinity: There exists a set having ∅ as element, as well as the successor of each of its elements.

پیدایش منطق نمادین در قرن ۱۹

- **Augustus de Morgan (1806-1871)**
- **George Boole (1815-1864)**
- **Charles Sanders Peirce (1839-1914)**
- **Ernst Schröder (1841-1902)**
- **F. L. Gottlob Frege (1848-1925)**

آثار مورد بحث فرگه

- ***Begriffsschrift*** (1879)

(مفهوم نگاری)

- ***Grundlagen der Arithmetik*** (1884)

(مبانی حساب)

- ***Grundgesetze der Arithmetik, I & II*** (1893, 1903)

(قوانین بنیادی حساب)

منطق فرگه

I. Language: variables x,y, …; connectives ¬, ∧, ∨, →, ↔;
predicates (= concepts) F, G, … ; =

II. First order quantifiers: $\exists x, \forall x, \ldots$

III. Identity: x=x

IV. Second order quantifiers: $\exists F, \forall F, \ldots$

V. (i) One-one correspondence of predicates and extensions (=classes = sets)

$$F \Leftrightarrow f : \quad f = \{x: Fx\} \text{ or } Fx \leftrightarrow x \in f$$

(ii) Identity of extensions: $f = g \Leftrightarrow Fx \leftrightarrow Gx$

+ Rules of deduction

قطعاتی از ***مبانی حساب*** فرگه (ترجمه J.L.Austin) :

- Empirical propositions hold good of what is physically or psychologically actual, the truths of geometry govern all that is spatially intuitable, whether actual or product of our fancy. …If we use of intuition even here, as an aid, it is still the same old intuition of Euclidean space. … Only then the intuition is not taken at its face value, but as symbolic of something else. … For purposes of conceptual thought, we can always assume the contrary of some one or the other of the geometrical axioms … this shows that the axioms of geometry are independent of one another and of the primitive laws of logic … (cont.)

قطعاتی از ***مبانی حساب*** فرگه (ترجمه J.L.Austin) :

- Can the same said of the fundamental propositions of the science of number? Here we only have to try denying any one of them, and complete confusion ensues. … The basis for arithmetic lies deeper, it seems, than that of any empirical sciences, and even that of geometry. The truths of arithmetic govern all that is numerable. This is the widest domain of all; for to it belongs not only the actual, not only the intuitable, but everything thinkable. Should not the laws of number, then, be connected very intimately with the laws of thought?

  (pp.20-21)

قطعاتی از ***مبانی حساب*** فرگه (ترجمه J.L.Austin) :

- … I may claim in the present work to have made it probable that the laws of arithmetic are analytic judgements and consequently a priori. Arithmetic thus becomes simply a development of logic, and every proposition of arithmetic a law of logic, albeit a derivative one. To apply arithmetic in the physical sciences is to bring logic to bear on observed facts; calculation becomes a deduction. The laws of number … need not to stand up to practical tests if they are to be applicable … for in the external world … there are no concepts …The laws of number, therefore, are not applicable to external things; they are not laws of nature. They are … applicable to judgements … they are laws of laws of nature. (p.99)

منطق‌گرایی به روایت راسل–وایتهد

- Russell, B. ***The Principles of Mathematics***, 1903
- ---------- ***Introduction to Mathematical Philosophy***, 1919
- ---------- & A. N. Whitehead ***Principia Mathematica*** (3 vols.), 1910-13, 1925-27

- نظریه انواع
- اصل بینهایت
- نظریه شاخه‌دار انواع
- اصل فروکاهی

تزلزل عرصه «وجود» در ریاضیات

- جایگاه هندسه
- دستگاه‌های عددی و جبری مجرد
- مجموعه‌ها، اعداد ترامتناهی
- جستجو برای مبانی وجود (؟)
- جستجو برای مبانی قطعیت (؟)

دگرگونی عرصه «وجود» و بحران قطعیت

- اصول تجرید فرگه، رابطه هم‌ارزی
- نظریه مجموعه‌ها، اعداد ترامتناهی
- انتقال مبحث‌های وجود و قطعیت ریاضیات از علوم طبیعی، متافیزیک والهیات به منطق، زبان و مجموعه‌ها
- شیوع پارادکس
- پیدایش عکس‌العمل به این گرایش‌ها و رویدادها

ریشه‌های پیدایش ساخت‌گرایی

- پیشینه «ساختن» از اقلیدس تا کانت
- دغدغه بینهایت
- طرد شق ثالث
- توسیع ناموجه استفاده از زبان
- تمایلات جاری حسابی‌سازی ریاضیات
- ارتباط ریاضیات با واقعیات (معنی‌دار بودن ریاضیات)

مکتب‌های ریاضیات ساختی

- **Kronecker**
- **Brouwer, Heyting, … (Intuitionism)**
- **Markov, recursive function theory, …**
- **Bishop’s constructive analysis**
- **Martin-Löf’s Type Theory, …**

L. E. J. Brouwer (1881-1966)

Selections in the following collections:

- J. van Heijenoort ***From Frege to Gödel***, 1967
- W. Ewald ***From Kant to Hilbert*** (Volume II), 1996
- P. Mancosu ***From Brouwer to Hilbert***, 1998

D. Van Dalen ***Brouwer's Cambridge lectures on intuitionism***, 1983

`The Nature of Geometry,' in L. E. J. Brouwer ***Collected Works*** (Vol.1), 1975

Errett Bishop (1928-1983)

- ***Constructive Analysis*** (with D. Bridges), 1985
  Revision of Bishop's *Foundations of Constructive Analysis*, 1967
  Chapter 1: A Constructivist Manifesto

- `The Crisis in Contemporary Mathematics' in
  Historia Mathematica 2(1975), 505-517

- `Mathematics as a Numerical Language' in
  Intuitionism and Proof Theory (eds. Kino, Myhill and Vesley), 1970

نقل از مقاله «بحران در ریاضیات معاصر» بیشاپ

- There is only one basic criterion to justify the philosophy of mathematics, and that is, does it contribute to making mathematics more meaningful. It is not true that this criterion is commonly accepted. In fact, the philosophical criterion that most mathematicians prefer is that it enables them to prove more theorems and to be more secure about the theorems that they have already proved.

David Hilbert (1862-1943)

Selections in the following collections:

- J. van Heijenoort ***From Frege to Gödel***, 1967
- W. Ewald ***From Kant to Hilbert*** (Volume II), 1996
- P. Mancosu ***From Brouwer to Hilbert***, 1998

M. Detlefsen ***Hilbert's Program***, 1986

C. Franks ***The Autonomy of Mathematical Knowledge***, 2009

W. Sieg ***Hilbert's Program and Beyond***, 2013

Articles by P. Bernays, J. von Neumann, H. Curry and G. Kreisel

چرا صورتگرایی؟

- ***In the beginning was the Sign***

  Hilbert: The New Grounding of Mathematics:

  First Report, 1922

- ***In the beginning was the Word***

  - John’s Bible

قطعاتی از مقاله **درباره بینهایت**، ۱۹۲۵، هیلبرت (۱)

- … the definitive clarification of the *nature of the infinite* has become necessary, not merely for the special interests of the individual sciences, but rather for the *honor of human understanding* itself. The infinite has always stirred the *emotions* of mankind more deeply than any other question; the infinite has stimulated and fertilized reason as few other *ideas* have; but also, the infinite, more than any other *notion*, is in need of *clarification.*

قطعاتی از مقاله **درباره بینهایت**، ۱۹۲۵، هیلبرت (۲)

- But there is a completely satisfactory way of escaping the paradoxes without committing treason against our science …

(1) We shall carefully investigate those ways of forming notions and those modes of inference that are fruitful; we shall nurse them, support them, and make them usable, wherever there is the slightest promise of success. No one shall be able to drive us from the paradise that Cantor created for us.

قطعاتی از مقاله **درباره بینهایت**، ۱۹۲۵، هیلبرت (۳)

(2) It is necessary to make inferences everywhere as reliable as they are in ordinary elementary number theory, which no one questions and in which contradictions and paradoxes arise only through our carelessness.

Obviously, we shall be able to reach these goals only if we succeed in completely clarifying *the nature of the infinite*.

قطعاتی از مقاله **درباره بینهایت**، ۱۹۲۵، هیلبرت (۴)

- We saw earlier that the infinite is not to be found anywhere in reality, …
- And has the contentual logical inference ever deceived or abandoned us anywhere when we applied it to real objects …
- It has deceived us only when we accepted arbitrary abstract notions, in particular those under which infinitely many objects are subsumed …
- Kant already taught …that mathematics has at its disposal a content secured independently of all logic …

هیلبرت درباره رابطه ریاضیات با منطق و فلسفه

- …it is the consistency proof that determines the effective scope of my proof theory … Already at this time I should like to assert what the final outcome will be: mathematics is a presupposition-less science.

  - `The foundations of Mathematics,' 1927

  (van Heijenoort ***From Frege to Gödel***)

- I believe that in my proof theory I have fully attained what I desired and promised: The world has hereby been rid, once and for all, of the question of the foundations of mathematics as such.

  - `The grounding of elementary number theory,' 1931

  (Mancosu ***From Brouwer to Hilbert***)

Skolem's Primitive Recursive Arithmetic (PRA)

- Variables of language (countable): x, y, z, …
- Propositional connectives: ¬, ∧, …
- =, 0, S (successor)
- A symbol for each primitive recursive function
- Axioms:

  (1) Tautologies of propositional calculus

  (2) S(x)≠ 0, $S(x) = S(y) \rightarrow x = y$

  (3) Arithmetical axioms: $x+0=x$, $x \cdot S(y) = x \cdot y + x$, …
- $\varphi(0) \wedge (\varphi(x) \rightarrow \varphi(S(x))) \rightarrow \varphi(y)$ for each predicate φ

تاملات هیلبرت در مورد هسته شهودی ریاضیات

- …the common prejudice that mathematics is but a continuation, a further development, of the fine art of arithmetic, of juggling with numbers. Our book aims to combat this prejudice …
  Hilbert and Cohn-Vossen ***Geometry and Imagination***

- We have to extend the domain of objects to be considered; i.e., we have to apply our intuitive considerations also to figures that are not number signs. Thus, we have good reason to distance ourselves from the earlier dominant principle according to which each theorem of pure mathematics is in the end a statement concerning integers… This method was viewed as expressing a fundamental methodological insight, but it has to be given up as a prejudice.
Hilbert-Bernays Lectures, 1921-22

قضایای گودل (۱۹۳۱)

قضیه اول هردستگاه صوری سازگار T که شامل «مقداری» حساب مقدماتی باشد ناتمام است ، یعنی حکمی در T قابل بیان است که نه خود آن اثبات‌پذیر است و نه نفی آن.

قضیه دوم در هردستگاه صوری سازگار T که شامل «مقداری» حساب مقدماتی باشد، سازگاریT درآن اثبات‌پذیر نیست.

مراجعی برای بحث پیرامون قضایای گودل

- Boolos, (Burgess) and Jeffrey ***Computability and Logic***, 1974-2007
- T. Franzen ***Gödel's Theorem: An Incomplete Guide to Its Use and Abuse***, 2005
- P. Raatikainen `Gödel's Incompleteness Theorems,' in ***Stanford Encyclopedia of Philosophy***, 2020
- P. Smith ***An Introduction to Gödel's Theorems***, 2007
- R. Smullyan ***Gödel's Incompleteness Theorems***, 1992

در حاشیه قضیه گودل

- احکام گلدباخ‌مانند
- قضیه گودستاین
- قضیه کروسکال
- قضیه آخر فرما
- فراقضیه‌ها

درباره اثبات قضیه آخر فرما

- What does it take to prove Fermat's Last Theorem? Grothendieck and the logic of number theory

  by Colin McLarty

  The Bulletin of Symbolic Logic, Vol. 16, No. 3,

  September 2010, pp 359-377

Richard Rorty “Keeping Philosophy Pure”

- In every generation, brilliant and feckless philosophical naifs … turn from their own specialties to expose the barrenness of academic philosophy and to explain how some or all of the old philosophical problems will yield to insights gained outside philosophy – only to have philosophy professors wearily explain that nothing has changed at all.

  - Consequences of Pragmatism, 1982

چندمرجع برای فلسفه ریاضی معاصر

- از فهرست Bostock, Colyvan, Linnebo and Shapiro : A
- از فهرست Hart, Irvine and Shapiro : B
- مقاله «**گرایش‌های موجود در فلسفه ریاضیات**» از حمید وحید ***نشر ریاضی***، سال ۱۰، شماره ۲ (۱۳۷۸)، ۷-۲۰
- مقاله «**افسانه‌گرایی و استعاره در فلسفه ریاضیات**» از سیاوش شهشهانی ***فرهنگ و اندیشه ریاضی***، شماره ۶۱ (پاییز و زمستان ۱۳۹۶)، ۱-۱۵

Imre Lakatos (1922-1974)

- ***Proofs and Refutations: The Logic of Mathematical Discovery***, 1976
- ***The Methodology of Scientific Research Programs*** (ed. J. Worrall and G. Currie), 1978
- ***Mathematics, Science and Epistemology*** (ed. J. Worrall and G. Currie), 1978
- ***Criticism and the Growth of Knowledge*** (ed. I. Lakatos and A. Musgrave), 1970
- T. Koestier ***Lakatos Philosophy of Mathematics***, 1991

تاثیر پوپر و پولیا بر لاکاتش

- Karl Popper:

  The Logic of Scientific Discovery, 1934

  Conjectures and Refutations, 1963

- George Pólya:

رویکرد شبه تجربی اکتشافی در ریاضیات

Mathematics and Plausible Reasoning (2 volumes), 1954

How to Solve It, 1945 (2004)

(Method of Heuristics)

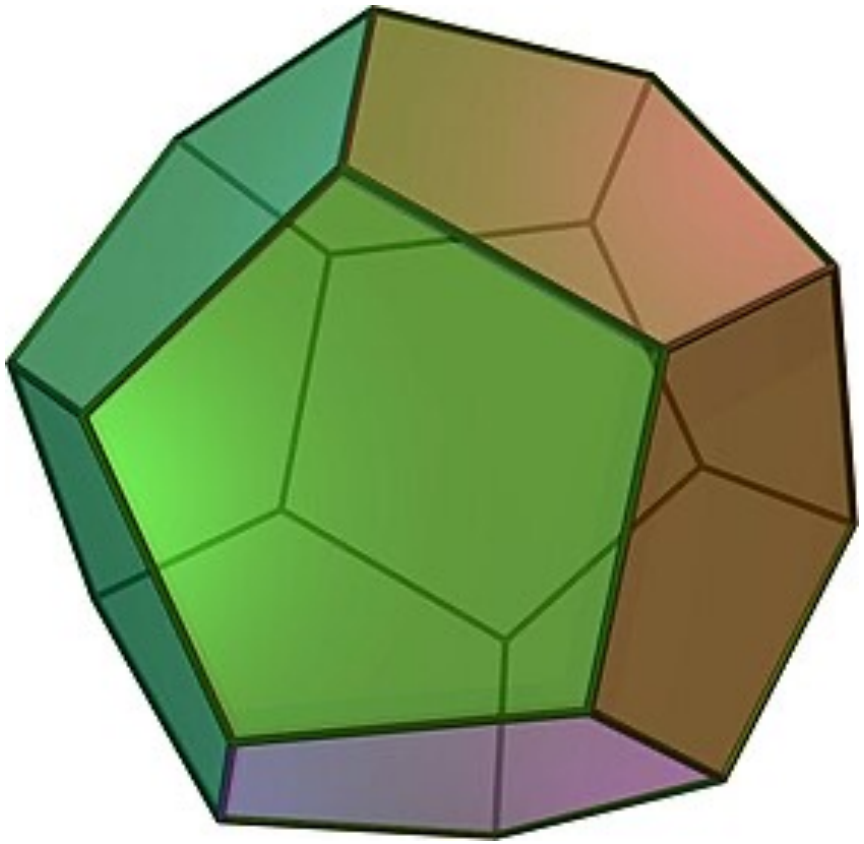
from *Proofs and Refutations*, p.144

… deductivist style tears the proof-generated definitions off their `proof ancestors', presents them out of the blue, in an artificial and authoritarian way. It hides the global counterexamples which led to their discovery. Heuristic style on the contrary highlights these factors. It emphasizes the problem situation: it emphasizes the `logic' that gave birth to the new concept.

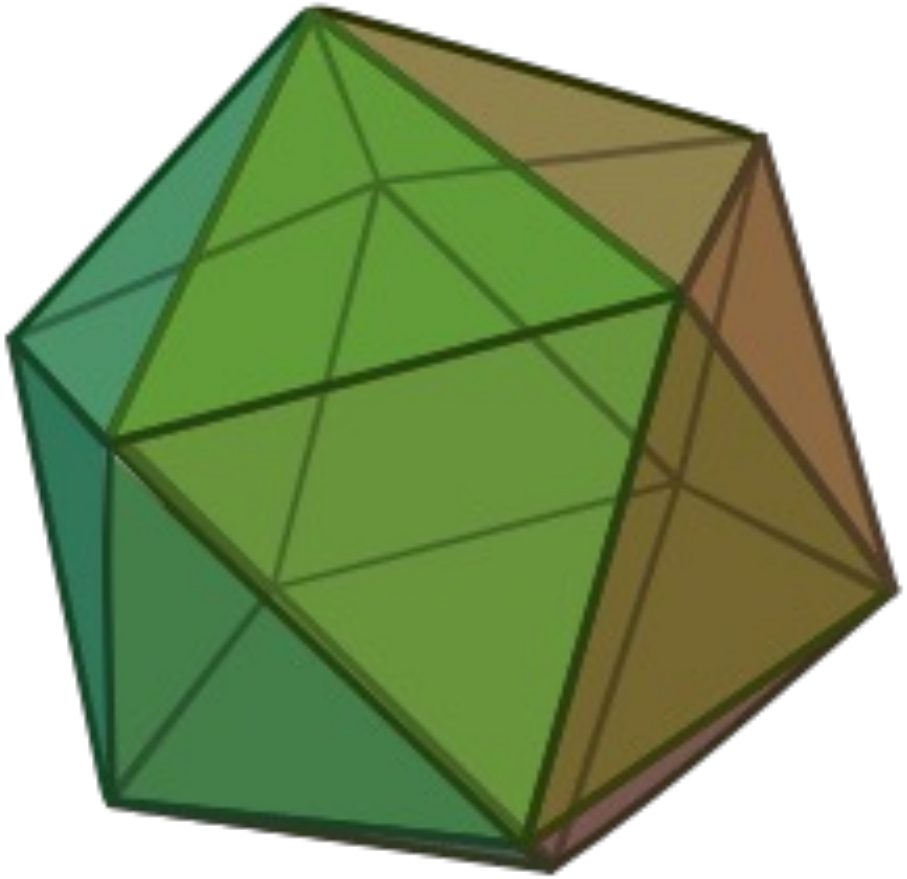
from *Proofs and Refutations*, p.146

Mathematical activity is a human activity. Certain aspects of this activity – as of any human activity – can be studied by psychology, others by history. Heuristic is not primarily interested in these aspects. But mathematical activity produces mathematics. Mathematics, this product of human activity, `alienates itself' from the human activity which has been producing it. It becomes a living, growing organism, that acquires a certain autonomy from the activity which has produced it; it develops its own autonomous laws of growth, …

Dodecahedron: 20 - 30 + 12 = 2



Icosahedron: 12 – 30 + 20 = 2



اقوال چندفیزیک‌دان معاصر از رابطه ریاضیات و فیزیک

- E. Wigner `The Unreasonable Effectiveness of Mathematics in the Natural Sciences' *Communications in Pure and Applied Mathematics*, Vol.XIII, 1-14 (1960)
- F. Dyson `Missed Opportunities' *Bulletin Amer. Math. Soc.*, Vol.78, No. 5, September 1972
- S. Weinberg in `Mathematics: The Unifying Thread in Science' *Notices Amer. Math. Soc.*, Vol.33, No.5, 725-728 (October 1986)

چند اثر معاصر قابل تامل

- Yu. I. Manin ***Mathematics and Physics***, 1981
- ---------- ***Mathematics and Metaphor***, 2000
- J.-P. Changeux and A. Connes ***Conversations on Mind, Matter and Mathematics***, 1995
- A. Jaffe and F. Quinn `Theoretical Mathematics: Toward a cultural synthesis of mathematics and theoretical physics,' in *Bulletin Amer. Math. Soc.* 30(1), 1994. Responses: *Bulletin Amer. Math. Soc*. 30(2).

سه کتاب عمومی پوانکاره پیرامون ریاضیات، منطق و فیزیک

- ***Science and Hypothesis***, 1905
- ***The Value of Science***, 1913
- ***Science and Method***, 1014

تاریخ‌های بالا مربوط به نشر ترجمه انگلیسی کتاب‌ها است. سه کتاب در یک مجلد زیرنیز به چاپ رسیده است:

- ***The Value of Science: Essential Writings of Henri Poincaré***

Edited by Stephen J. Gould, 2001.

نمونه گرایش‌های متاخر

فیزیک	ریاضیات	
واقع‌گرا	واقع‌گرا	**پاتنم**
واقع‌گرا	نام‌گرا	**فیلد**
نام‌گرا	نام‌گرا	**فن فراسن**

Debate on Application of Mathematics

- H. Putnam **Philosophy of Logic**, 1971
- ---------- `What is Mathematical Truth?' in *Historica Mathematica,*2 (1975), 529-533. Also in

  Mathematics, Matter and Method, ***Philosophical Papers***, ***Vol.1***, 1975
- H. Field ***Science Without Numbers: A defence of nominalism***, 1980
- B. Van Fraassen ***The Scientific Image***, 1980

Realism: Classical and Modern

- In [the Aristotelian] tradition, the realists held that regularities in the natural phenomena must have a reason (cause, explanation), and they sought this reason in the causal properties, constituting what they called the substantial forms or natures, of the substances involved in the natural processes.

  - B. van Fraassen in *The Scientific Image*

- A realist, with respect to a given theory or discourse holds that (1) the sentences of that theory or discourse are true or false; and (2) that what makes them true or false is something *external*.

  - H. Putnam in `What is Mathematical Truth'

(Category and Topos Theory) نظریه رسته و توپوس

- S. Eilenberg and S. Mac Lane `General Theory of Natural Equivalences' *Transactions Amer.Math.Soc*. **58**: 247, 1954
- S. Eilenberg and N. Steenrod ***Foundations of Algebraic Topology***, 1952
- S. Eilenberg and H. Cartan ***Homological Algebra***, 1956
- A. Grothendieck and J.L. Verdier ***Théorie des Topos et Cohomologie Etale des Schémas*** Lecture notes in mathematics. Vol. 269, 1972
- Current definition of ***Topos*** due to W. Lawvere and M. Tierney

تعاریف اصلی رسته

- Object, arrow (morphism), composition and its two properties

  Examples
- Functors (covariant and contravariant)

  Examples (especially in topology to algebra)
- Natural transformations

  Examples

Axioms of Topos

1. ***Terminal object* 1**
2. ***Pullbacks***
3. ***Exponential***
4. ***Sub-object classifier***

(Elementary topos)

---

5. Axiom of Infinity

6. Axiom of Choice

……

علوم شناختی و ریاضیات

- J. Piaget ***The Child's Conception of Number***, 1952
- N. Chomsky ***Language and Mind*** (3rd edition), 2006
- S. Carey ***The Origin of Concepts***, 2009

- S. Carey and E. Spelke `Domain-specific knowledge and conceptual change' in ***Mapping the Mind*** (eds. L Hirschfeld and S. Gelman), 1994
- M.D.Hauser, N.Chomsky and W.T.Fitch `The Faculty of Language: What Is It, Who Has It, and How Did It Evolve?' in *Science* 298, 1569-1579, 22 Nov.2002

- An elementary fact about the language faculty is that it is a system of discrete infinity, rare in the organic world. Any such system is based on a primitive operation that takes objects already constructed, and constructs from them a new object: in the simplest case, the set containing them. Call that operation Merge.

  …

- With Merge available, we instantly have an unbounded system of expressions. The simplest account of the “Great Leap Forward” in the evolution of humans would be that the brain was rewired, perhaps by some slight mutation, to provide the operation Merge, at once laying a core part of the basis for what is found at that dramatic moment of human evolution …

- The most restrictive case of Merge … yields the successor function, from which the rest of the theory of natural numbers can be developed in familiar ways. That suggests a possible answer to a problem that troubled Wallace in the late nineteenth century: in his words, that the "gigantic development of the mathematical capacity is wholly unexplained by the theory of natural selection, and must be due to some altogether distinct cause,"…. One possibility is that the natural numbers result from a simple constraint on the language faculty …Speculations about the origin of the mathematical capacity as an abstraction from linguistic operations are not unfamiliar. There are apparent problems, including dissociation with lesions and diversity of localization, …

- Chomsky, *Language and Mind* (3rd ed), pp.183-4

- Why did humans, but no other animal, take the power of recursion to create an open-ended and limitless system of communication? Why does our system of recursion operate over a broader range of elements or inputs (e.g., numbers, words) than other animals? One possibility, consistent with current thinking in the cognitive sciences, is that recursion in animals represents a modular system designed for a particular function (e.g., navigation) and impenetrable with respect to other systems. During evolution, the modular and highly domain-specific system of recursion may have become penetrable and domain-general. This opened the way for humans, perhaps uniquely, to apply the power of recursion to other problems. This change from domain-specific to domain-general may have been guided by particular selective pressures, unique to our evolutionary past, or as a consequence (by-product) of other kinds of neural reorganization.

- Hauser, et al, *Science* 2002

(Core Cognition) شناخت درونگاهی

- Advocated by some cognitive scientists, notably Susan Carey and Elizabeth Spelke.
- A capacity between perception and conceptual thinking (similar to `*wahm*' enunciated by Ebne Sina)
- Learning devices developed early in life in specific modules and persisting permanently unlike conceptual beliefs
- Some are shared by other animals including certain object and behavioral recognitions
- Two capacities for dealing with small numbers seem to be core cognitions

Quotation from Carey's *Origin of Concepts* (1)

- … cognition of humans, like that of all animals, begins with highly structured innate mechanisms designed to build representations with specific content. I call these real-world content domains "core domains," and I call the mental structures that represent them "core cognition". ….core cognition has rich integrated conceptual content. By this I mean that the representations in core cognition cannot be reduced to perceptual or sensory-motor primitives,…

Quotation from Carey's *Origin of Concepts* (2)

- Core cognition is elaborated during development because core cognition systems are learning devices, but it is never rendered irrelevant. It is never overturned or lost, in contrast to later developing intuitive theories, which are sometimes replaced by subsequent, incommensurable ones….systems of core cognition are domain-specific learning devices some core cognition (including that of objects) is shared by other animals. At least some early developing cognitive systems in humans have a long evolutionary history…

علم مغز و ریاضیات

- S. Dehaene ***The Number Sense***, 1997, 2011
- B. Butterworth ***Mathematical Brain***, 1999

- S. Dehaene and J.-P. Changeux `Development of Elementary Numerical Abilities: A Neuronal Model,' *J. Cognitive Neuroscience* 5(1993):4, 390-407
- M. Amalric and S, Dehaene `Cortical circuits for mathematical Knowledge: evidence for a major subdivision within the brain's neural network', *Phil. Trans.R.Soc.B* 373: 2016.05.15, 1-9
- J. Kulasingham, et al `Cortical Processing of Arithmetic and Simple Sentences in an Auditory Attention Task,' *J.Neuroscience*, September 22, 2021; 41(38) 8023-8039

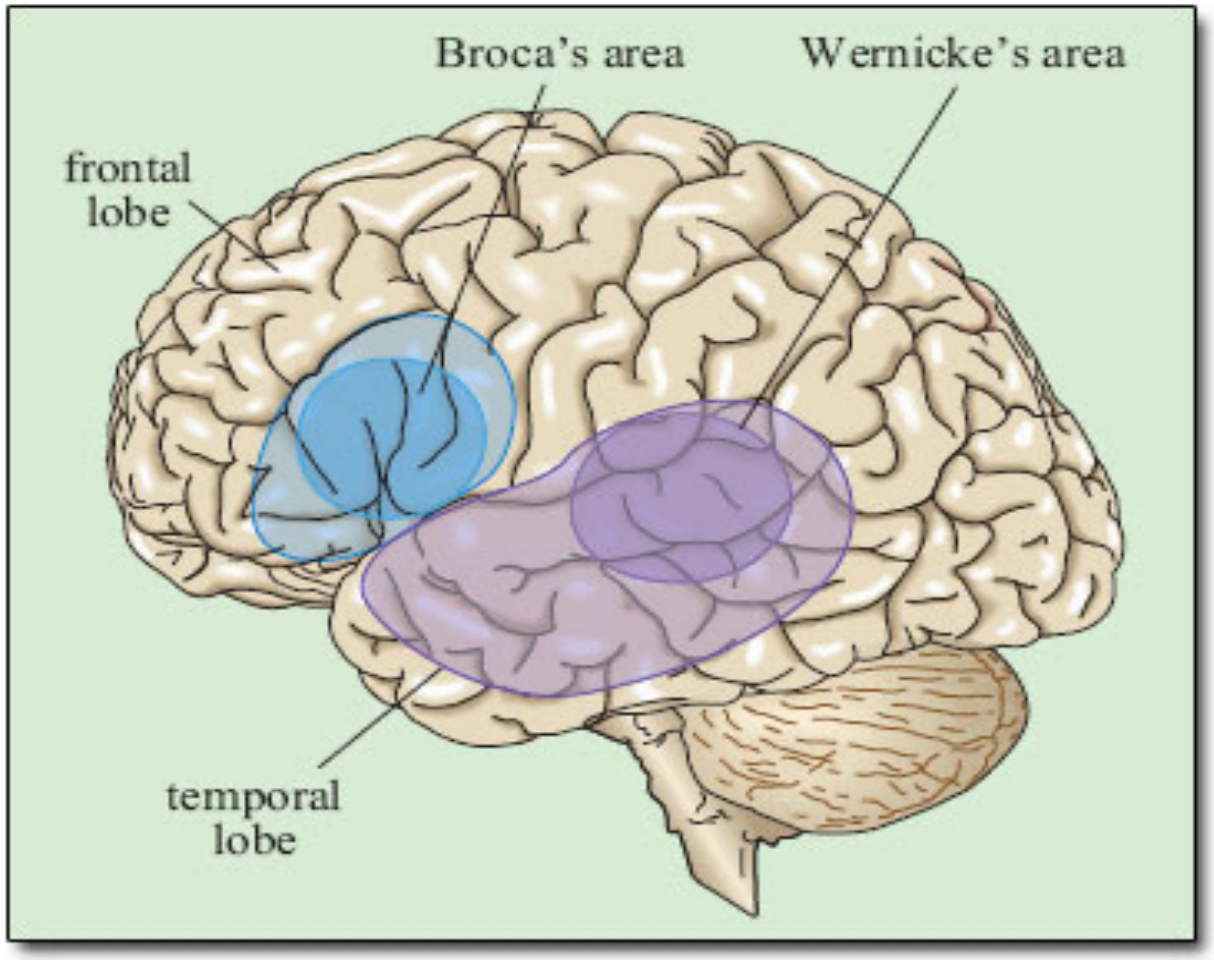
Language vs Mathematical Semantics

Is mathematical language similar to natural language? Are language areas used by mathematicians when they do mathematics? And does the brain comprise a generic semantic system that stores mathematical knowledge alongside knowledge of history, geography or famous people? Here, we refute those views by reviewing three functional MRI studies of the representation and manipulation of high-level mathematical knowledge in professional mathematicians …

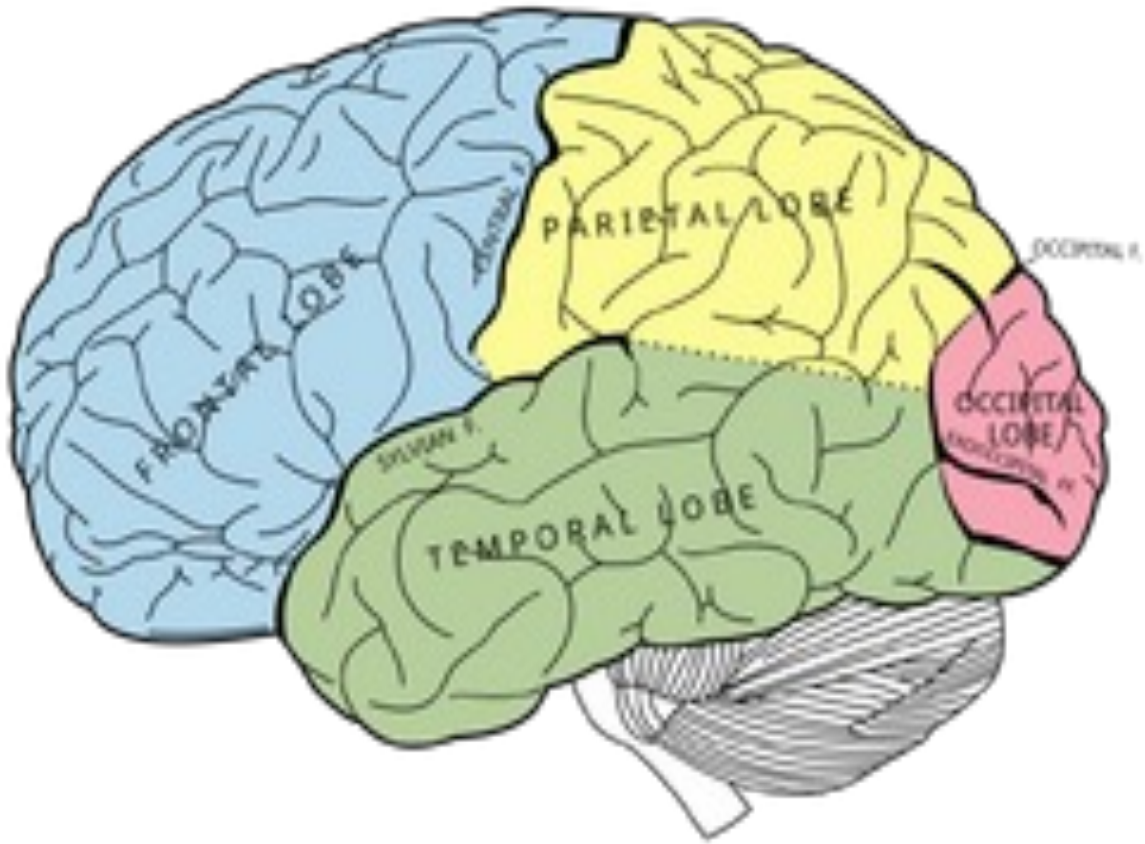
… brain activity during professional mathematical reflection spares perisylvian language-related brain regions as well as temporal lobe areas classically involved in general semantic knowledge. Instead, mathematical reflection recycles bilateral intraparietal and ventral temporal regions involved in elementary number sense. Even simple fact retrieval, such as remembering that ‘the sine function is periodical’ or that ‘London buses are red’, activates dissociated areas for math versus non-math knowledge. Together with other fMRI and recent intracranial studies, our results indicated a major separation between two brain networks for mathematical and non-mathematical semantics.

- M.Amalric and S. Dehaene

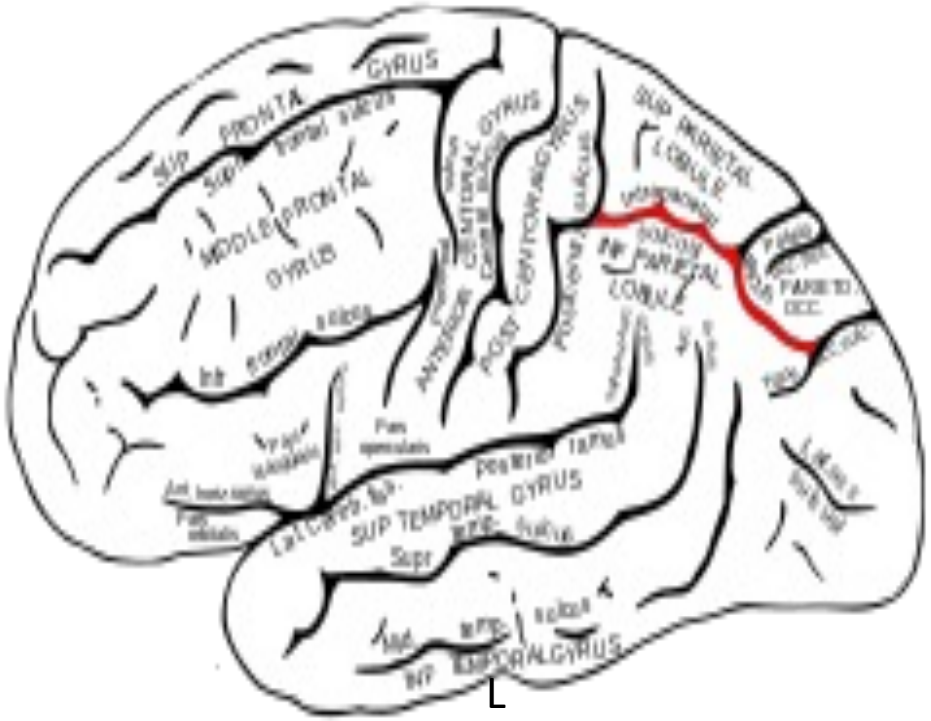
Language and General Semantic Regions

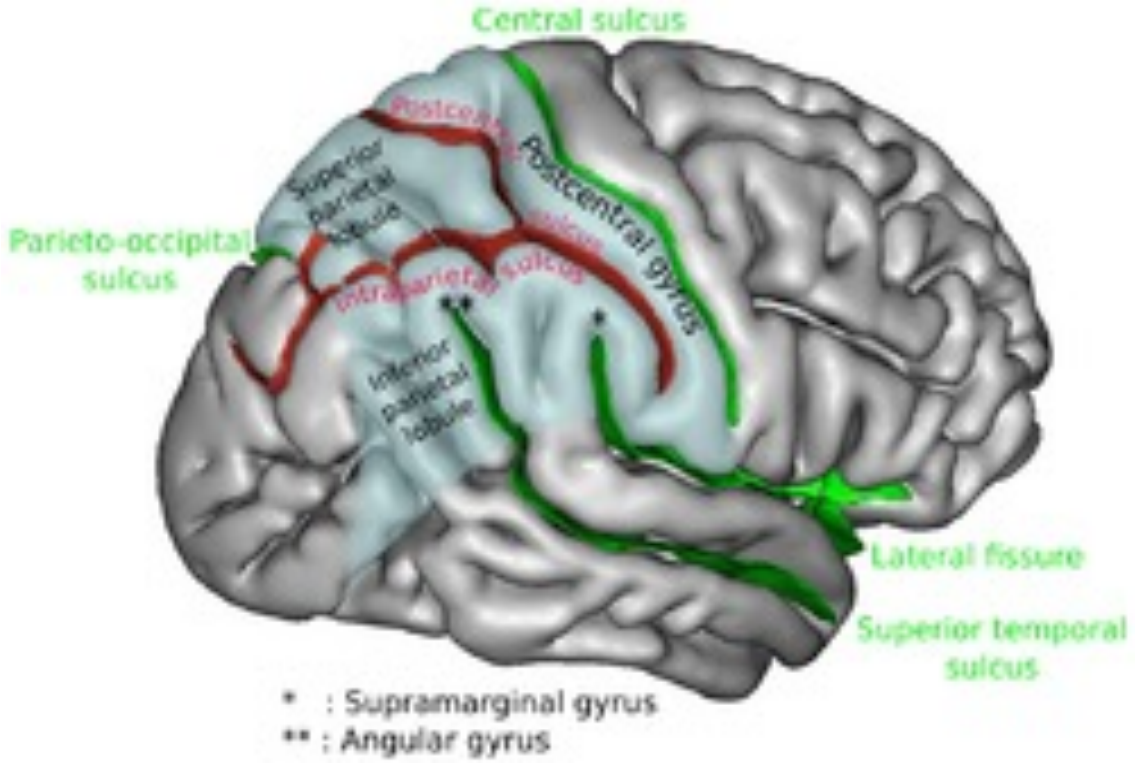


Parietal Lobe



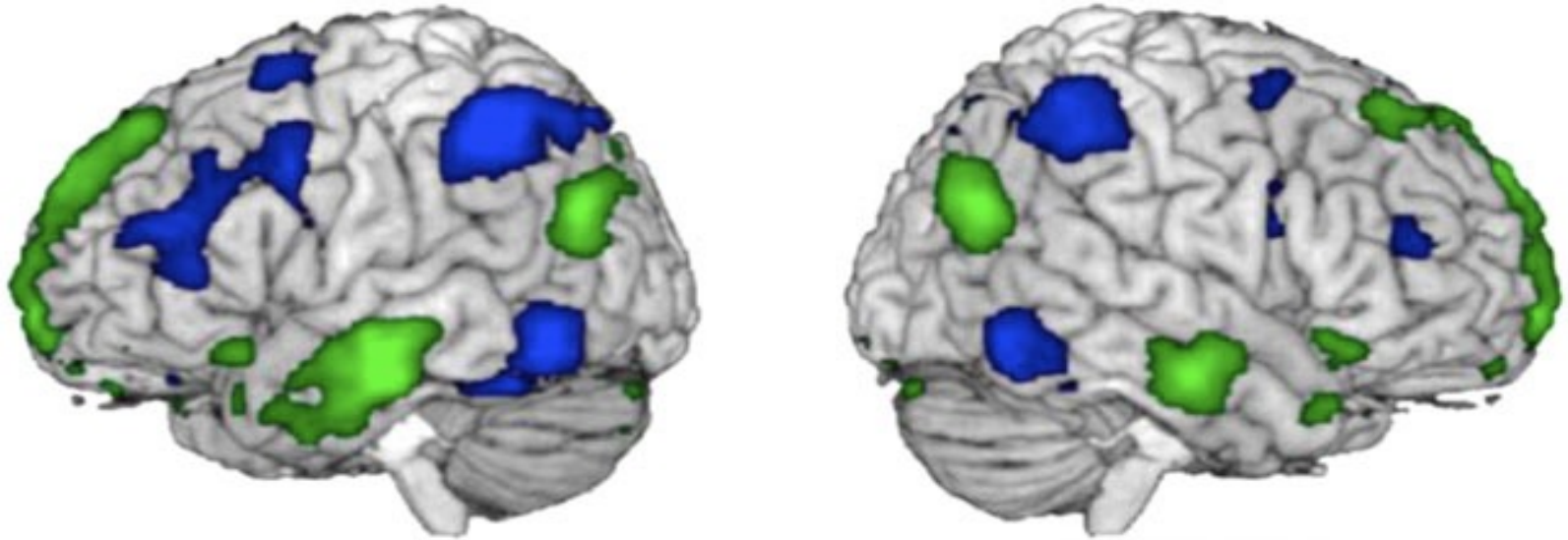
Intraparietal sulcus





Response of Mathematicians' Brains

'Lp spaces are separable' versus 'The Paris metro was built before the Istanbul one'



math > nonmath nonmath > math

MEG Confirmation