هیلبرت درباره رابطه ریاضیات با منطق و فلسفه

- …it is the consistency proof that determines the effective scope of my proof theory … Already at this time I should like to assert what the final outcome will be: mathematics is a presupposition-less science.

  - `The foundations of Mathematics,' 1927

    (van Heijenoort *From Frege to Gödel*)

- I believe that in my proof theory I have fully attained what I desired and promised: The world has hereby been rid, once and for all, of the question of the foundations of mathematics as such.

  - `The grounding of elementary number theory,' 1931

    (Mancosu *From Brouwer to Hilbert*)

Skolem's Primitive Recursive Arithmetic (PRA)

- Variables of language (countable): x, y, z, …

- Propositional connectives: $\neg$, $\wedge$, …

- =, 0, S (successor)

- A symbol for each primitive recursive function

- Axioms:

  (1) Tautologies of propositional calculus

  (2) S(x)$\neq$ 0, $S(x) = S(y) \rightarrow x = y$

  (3) Arithmetical axioms: *x+0=x*, *x·S(y) = x·y + x, …*

- $\varphi(0) \wedge (\varphi(x) \rightarrow \varphi(S(x))) \rightarrow \varphi(y)$ for each predicate φ

تاملات هیلبرت در مورد هسته شهودی ریاضیات

- …the common prejudice that mathematics is but a continuation, a further development, of the fine art of arithmetic, of juggling with numbers. Our book aims to combat this prejudice …
   Hilbert and Cohn-Vossen ***Geometry and Imagination***

- We have to extend the domain of objects to be considered; i.e., we have to apply our intuitive considerations also to figures that are not number signs. Thus, we have good reason to distance ourselves from the earlier dominant principle according to which each theorem of pure mathematics is in the end a statement concerning integers… This method was viewed as expressing a fundamental methodological insight, but it has to be given up as a prejudice.
Hilbert-Bernays Lectures, 1921-22

قضایای گودل (۱۹۳۱)

قضیه اول هردستگاه صوری سازگار T که شامل «مقداری» حساب مقدماتی باشد ناتمام است ، یعنی حکمی در T قابل بیان است که نه خود آن اثبات‌پذیر است و نه نفی آن.

قضیه دوم درهردستگاه صوری سازگار T که شامل «مقداری» حساب مقدماتی باشد، سازگاریT درآن اثبات‌پذیر نیست.

مراجعی برای بحث پیرامون قضایای گودل

- Boolos, (Burgess) and Jeffrey *Computability and Logic*, 1974-2007

- T. Franzen *Gödel's Theorem: An Incomplete Guide to Its Use and Abuse*, 2005

- P. Raatikainen `Gödel's Incompleteness Theorems,' in *Stanford Encyclopedia of Philosophy*, 2020

- P. Smith *An Introduction to Gödel's Theorems*, 2007

- R. Smullyan *Gödel's Incompleteness Theorems*, 1992

در حاشیه قضیه گودل

- احکام گلدباخ‌مانند
- قضیه گودستاین
- قضیه کروسکال
- قضیه آخر فرما
- فراقضیه‌ها

درباره اثبات قضیه آخر فرما

- What does it take to prove Fermat's Last Theorem? Grothendieck and the logic of number theory

  by Colin McLarty

  The Bulletin of Symbolic Logic, Vol. 16, No. 3,

  September 2010, pp 359-377

Richard Rorty "Keeping Philosophy Pure"

- In every generation, brilliant and feckless philosophical naifs … turn from their own specialties to expose the barrenness of academic philosophy and to explain how some or all of the old philosophical problems will yield to insights gained outside philosophy – only to have philosophy professors wearily explain that nothing has changed at all.

  - Consequences of Pragmatism, 1982