

# تزلزل عرصه « وجود » در ریاضیات

- جایگاه هندسه
- دستگاه‌های عددی و جبری مجرد
- مجموعه‌ها، اعداد ترامتناهی
- جستجو برای مبانی وجود (؟)
- جستجو برای مبانی قطعیت (؟)

# دگرگونی عرصه « وجود » و بحران قطعیت

- اصول تجرید فرگه، رابطه هم‌ارزی
- نظریه مجموعه‌ها، اعداد ترامتناهی
- انتقال مباحث‌های وجود و قطعیت ریاضیات از علوم طبیعی، متافیزیک و الهیات به منطق، زبان و مجموعه‌ها
- شیوع پارادکس
- پیدایش عکس‌العمل به این گرایش‌ها و رویدادها

# ریشه‌های پیدایش ساخت گرای

- پیشینه « ساختن » از اقلیدس تا کانت
- دغدغه بینهایت
- طرد شق ثالث
- توسعه ناموجه استفاده از زبان
- تمایلات جاری حسابی سازی ریاضیات
- ارتباط ریاضیات با واقعیات ( معنی دار بودن ریاضیات )

# مکتب‌های ریاضیات ساختی

- **Kronecker**
- **Brouwer, Heyting, ... (Intuitionism)**
- **Markov, recursive function theory, ...**
- **Bishop's constructive analysis**
- **Martin-Löf's Type Theory, ...**

# L. E. J. Brouwer (1881-1966)

Selections in the following collections:

- J. van Heijenoort *From Frege to Gödel*, 1967
- W. Ewald *From Kant to Hilbert* (Volume II), 1996
- P. Mancosu *From Brouwer to Hilbert*, 1998

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D. Van Dalen *Brouwer's Cambridge lectures on intuitionism*, 1983

'The Nature of Geometry,' in L. E. J. Brouwer *Collected Works* (Vol.1), 1975

# Errett Bishop (1928-1983)

- ***Constructive Analysis*** (with D. Bridges), 1985  
Revision of Bishop's *Foundations of Constructive Analysis*, 1967  
Chapter 1: A Constructivist Manifesto
- 'The Crisis in Contemporary Mathematics' in  
*Historia Mathematica* 2(1975), 505-517
- 'Mathematics as a Numerical Language' in  
***Intuitionism and Proof Theory*** (eds. Kino, Myhill and Vesley), 1970

## نقل از مقاله «بحران در ریاضیات معاصر» بیشاپ

- There is only one basic criterion to justify the philosophy of mathematics, and that is, does it contribute to making mathematics more meaningful. It is not true that this criterion is commonly accepted. In fact, the philosophical criterion that most mathematicians prefer is that it enables them to prove more theorems and to be more secure about the theorems that they have already proved.

# David Hilbert (1862-1943)

Selections in the following collections:

- J. van Heijenoort ***From Frege to Gödel***, 1967
- W. Ewald ***From Kant to Hilbert*** (Volume II), 1996
- P. Mancosu ***From Brouwer to Hilbert***, 1998

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M. Detlefsen ***Hilbert's Program***, 1986

C. Franks ***The Autonomy of Mathematical Knowledge***, 2009

W. Sieg ***Hilbert's Program and Beyond***, 2013

Articles by P. Bernays, J. von Neumann, H. Curry and G. Kreisel



# چرا صورتگرایی؟

- ***In the beginning was the Sign***

Hilbert: The New Grounding of Mathematics:  
First Report, 1922

- ***In the beginning was the Word***

- John's Bible

قطعاتی از مقاله درباره بینهایت، ۱۹۲۵، هیلبرت (۱)

- ... the definitive clarification of the *nature of the infinite* has become necessary, not merely for the special interests of the individual sciences, but rather for the *honor of human understanding* itself. The infinite has always stirred the *emotions* of mankind more deeply than any other question; the infinite has stimulated and fertilized reason as few other *ideas* have; but also, the infinite, more than any other *notion*, is in need of *clarification*.

قطعاتی از مقاله درباره بینهایت، ۱۹۲۵، هیلبرت (۲)

- But there is a completely satisfactory way of escaping the paradoxes without committing treason against our science ...

(1) We shall carefully investigate those ways of forming notions and those modes of inference that are fruitful; we shall nurse them, support them, and make them usable, wherever there is the slightest promise of success. **No one shall be able to drive us from the paradise that Cantor created for us.**

قطعاتی از مقاله درباره بینهایت، ۱۹۲۵، هیلبرت (۳)

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(2) It is necessary to make inferences everywhere as reliable as they are in ordinary elementary number theory, which no one questions and in which contradictions and paradoxes arise only through our carelessness.

Obviously, we shall be able to reach these goals only if we succeed in completely clarifying *the nature of the infinite*.

## قطعاتی از مقاله درباره بینهایت، ۱۹۲۵، هیلبرت (۴)

- We saw earlier that the infinite is not to be found anywhere in reality, ...
- And has the contentual logical inference ever deceived or abandoned us anywhere when we applied it to real objects ...
- It has deceived us only when we accepted arbitrary abstract notions, in particular those under which infinitely many objects are subsumed ...
- Kant already taught ...that mathematics has at its disposal a content secured independently of all logic ...

## هیلبرت درباره رابطه ریاضیات با منطق و فلسفه

- ...it is the consistency proof that determines the effective scope of my proof theory ... Already at this time I should like to assert what the final outcome will be: mathematics is a presupposition-less science.
  - 'The foundations of Mathematics,' 1927  
(van Heijenoort *From Frege to Gödel*)
- I believe that in my proof theory I have fully attained what I desired and promised: The world has hereby been rid, once and for all, of the question of the foundations of mathematics as such.
  - 'The grounding of elementary number theory,' 1931  
(Mancosu *From Brouwer to Hilbert*)