Axiomatic Set Theory

- Ernst Zermelo, 1908
- Zermelo-Fraenkel, 1921, contributions by Skolem and von Neumann
- Formulated in first order logic. Undefined: Set, ∈
- ZFC = ZF + Axiom of Choice or equivalent

- Axiom of Extensionality: Sets are determined by their extensions
- Axiom of Regularity: Every non-empty set S has an element x so that S and x have empty intersection. (Together with Axiom of Pairing and extensionality bans paradoxes involving x ∈ x ; let S = {x}.)
- Axiom of Infinity: There exists a set having $\emptyset$ as element, as well as the successor of each of its elements.

پیدایش منطق نمادین در قرن ۱۹

- **Augustus de Morgan (1806-1871)**
- **George Boole (1815-1864)**
- **Charles Sanders Peirce (1839-1914)**
- **Ernst Schröder (1841-1902)**
- **F. L. Gottlob Frege (1848-1925)**

آثار مورد بحث فرگه

- ***Begriffsschrift*** (1879)
(مفهوم نگاری)
- ***Grundlagen der Arithmetik*** (1884)
(مبانی حساب)
- ***Grundgesetze der Arithmetik, I & II*** (1893, 1903)
(قوانین بنیادی حساب)

منطق فرگه

I. <u>Language</u>: variables x,y, …; connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$;

predicates (= concepts) F, G, … ; =

II. <u>First order quantifiers</u>: $\exists x$, $\forall x$, ...

III. <u>Identity</u>: x=x

IV. <u>Second order quantifiers</u>: $\exists F$, $\forall F$, ...

V. (i) One-one correspondence of predicates and extensions (=classes = sets)

$$F \Leftrightarrow f: \quad f = \{x: Fx\} \text{ or } Fx \leftrightarrow x \in f$$

(ii) Identity of extensions: $f = g \Leftrightarrow Fx \leftrightarrow Gx$

+ Rules of deduction

قطعاتی از مبانی حساب فرگه (ترجمه J.L.Austin) :

- Empirical propositions hold good of what is physically or psychologically actual, the truths of geometry govern all that is spatially intuitable, whether actual or product of our fancy. …If we use of intuition even here, as an aid, it is still the same old intuition of Euclidean space. … Only then the intuition is not taken at its face value, but as symbolic of something else. … For purposes of conceptual thought, we can always assume the contrary of some one or the other of the geometrical axioms … this shows that the axioms of geometry are independent of one another and of the primitive laws of logic … (cont.)

قطعاتی از **مبانی حساب** فرگه (ترجمه J.L.Austin) :

- Can the same said of the fundamental propositions of the science of number? Here we only have to try denying any one of them, and complete confusion ensues. … The basis for arithmetic lies deeper, it seems, than that of any empirical sciences, and even that of geometry. The truths of arithmetic govern all that is numerable. This is the widest domain of all; for to it belongs not only the actual, not only the intuitable, but everything thinkable. Should not the laws of number, then, be connected very intimately with the laws of thought?

(pp.20-21)

قطعاتی از مبانی حساب فرگه (ترجمه J.L.Austin) :

- … I may claim in the present work to have made it probable that the laws of arithmetic are analytic judgements and consequently a priori. Arithmetic thus becomes simply a development of logic, and every proposition of arithmetic a law of logic, albeit a derivative one. To apply arithmetic in the physical sciences is to bring logic to bear on observed facts; calculation becomes a deduction. The laws of number … need not to stand up to practical tests if they are to be applicable … for in the external world … there are no concepts …The laws of number, therefore, are not applicable to external things; they are not laws of nature. They are … applicable to judgements … they are laws of laws of nature. (p.99)

منطق‌گرایی به روایت راسل–وایتهد

- Russell, B. ***The Principles of Mathematics***, 1903
- ------------- ***Introduction to Mathematical Philosophy***, 1919
- ------------- & A. N. Whitehead ***Principia Mathematica*** (3 vols.), 1910-13, 1925-27

- نظریه انواع
- اصل بینهایت
- نظریه شاخه‌دار انواع
- اصل فروکاهی