Game Theory - Week 3

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Overview

- Strictly Dominated Strategies & Iterative Removal
- Dominated Strategies & Iterative Removal:An Application
- Maxmin Strategies
- Correlated Equilibrium

Rationality

- A basic premise: players maximize their payoffs
- What if all players know this?
- And they know that other players know it?
- And they know that other players know that they know it?
- ...

Strictly Dominated Strategies

- A strictly dominated strategy can never be a best reply.
- Let us remove it as it will not be played.
- All players know this - so let us iterate...
- Running this process to termination is called the **iterated removal of strictly dominated strategies**.

Strictly Dominated Strategies (Definitions)

Definition (Strictly Dominated Strategies)

A strategy $s_i \in S_i$ is strictly dominated by $s_i' \in S_i$ (strategy profile $S = (s_1, \ldots, s_n)$) if

$$u_i(s_i, s_{-i}) < u_i(s_i', s_{-i}) \qquad \forall s_{-i} \in S_{-i}$$

Iterated Removal of Strictly Dominated Strategies: Example

	L	C	R
U	3, 0	2, 1	0, 0
M	1, 1	1, 1	5, 0
D	0, 1	4, 2	0, 1

Iterated Removal of Strictly Dominated Strategies: Example

	L	C	R
U	$3, 0$	$2, 1$	$0, 0$
M	$1, 1$	$1, 1$	$5, 0$
D	$0, 1$	$4, 2$	$0, 1$

- R is strictly dominated by C

Iterated Removal of Strictly Dominated Strategies - Example (cont'd)

	L	C
U	3, 0	2, 1
M	1, 1	1, 1
D	0, 1	4, 2

Iterated Removal of Strictly Dominated Strategies - Example (cont'd)

	L	C
U	$3, 0$	$2, 1$
M	$1, 1$	$1, 1$
D	$0, 1$	$4, 2$

- M is strictly dominated by U

Iterated Removal of Strictly Dominated Strategies: Example (cont'd)

	L	C
U	$3, 0$	$2, 1$
D	$0, 1$	$4, 2$

Iterated Removal of Strictly Dominated Strategies: Example (cont'd)

	L	C
U	$3, 0$	$2, 1$
D	$0, 1$	$4, 2$

- L is strictly dominated by C

Iterated Removal of Strictly Dominated Strategies: Example (cont'd)

	C
U	$2, 1$
D	$4, 2$

Iterated Removal of Strictly Dominated Strategies: Example (cont'd)

	C
U	$2, 1$
D	$4, 2$

- U is strictly dominated by D

Iterated Removal of Strictly Dominated Strategies: Example

	L	C	R
U	3, 0	2, 1	0, 0
M	1, 1	1, 1	5, 0
D	0, 1	4, 2	0, 1

Iterated Removal of Strictly Dominated Strategies: Example

	L	C	R
U	$3, 0$	$2, 1$	$0, 0$
M	$1, 1$	$1, 1$	$5, 0$
D	$0, 1$	$4, 2$	$0, 1$

- A unique Nash equilibrium C, D

Iterated Removal of Strictly Dominated Strategies: Another Example

	L	C	R
U	3, 1	0, 1	0, 0
M	1, 1	1, 1	5, 0
D	0, 1	4, 1	0, 0

Iterated Removal of Strictly Dominated Strategies: Another Example

	L	C	R
U	3, 1	0, 1	0, 0
M	1, 1	1, 1	5, 0
D	0, 1	4, 1	0, 0

- R is dominated by L or C

Iterated Removal of Strictly Dominated Strategies: Another Example (cont'd)

	L	C
U	$3, 1$	$0, 1$
M	$1, 1$	$1, 1$
D	$0, 1$	$4, 1$

Iterated Removal of Strictly Dominated Strategies: Another Example (cont'd)

	L	C
U	$3, 1$	$0, 1$
M	$1, 1$	$1, 1$
D	$0, 1$	$4, 1$

- M is dominated by the mixed strategy that selects U and D with equal probability.

Iterated Removal of Strictly Dominated Strategies: Another Example (cont'd)

	L	C
U	$3, 1$	$0, 1$
M	$1, 1$	$1, 1$
D	$0, 1$	$4, 1$

- M is dominated by the mixed strategy that selects U and D with equal probability.
- Can use mixed strategies to define domination too!

Iterated Removal of Strictly Dominated Strategies: Another Example (cont'd)

	L	C
U	$3, 1$	$0, 1$
D	$0, 1$	$4, 1$

Iterated Removal of Strictly Dominated Strategies: Another Example (cont'd)

	L	C
U	$3, 1$	$0, 1$
D	$0, 1$	$4, 1$

- No other strategies are strictly dominated.
- What are the Nash Equilibria?

Iterated Removal of Strictly Dominated Strategies

- This process **preserves Nash equilibria**
  - It can be used as a **preprocessing step** before computing an equilibrium
  - Some games are solvable using this technique - those games are **dominance solvable**

Iterated Removal of Strictly Dominated Strategies

- This process **preserves Nash equilibria**
  - It can be used as a **preprocessing step** before computing an equilibrium
  - Some games are solvable using this technique - those games are **dominance solvable**

- What about the **order of removal** when there are multiple strictly dominated strategies?
  - doesn't matter

Weakly Dominated Strategies

Definition

A strategy $s_i \in S_i$ is weakly dominated by $s_i' \in S_i$ if

$u_i(s_i, s_{-i}) \leq u_i(s_i', s_{-i})$ for all $s_{-i} \in S_{-i}$, and

$u_i(s_i, s_{-i}) < u_i(s_i', s_{-i})$ for some $s_{-i} \in S_{-i}$

- Can remove them iteratively too, but:

Weakly Dominated Strategies

- They can be best replies.
- Order of removal can matter.
- At least one equilibrium preserved.

First-price Auction

- You have cool $50 million With all this cash on hand.
- An auction house is selling an Andy Warhol piece.
- The rules are that all interested parties must submit a written bid and whoever submits the highest bid wins the Warhol piece and pays a price equal to the bid. This known as the **first-price auction**.

First-price Auction (cont'd)

- The Warhol piece is worth \$400,000 to you.
- You've just learned that there is only one other bidder: your old college friend.
- The Warhol piece is worth \$300,000 to your college, and furthermore.
- The auctioneer announces that bids must be in increments of \$100,000 and that the minimum bid is \$100,000 and the maximum bid is \$500,000.

First-price Auction (cont'd)

- If the bids are equal, the auctioneer flips a coin to determine the winner (payoffs are in hundreds of thousands of dollar).
- For example, if you bid 3 and she bids 1, then you win the auction, pay a price of 3, and receive a payoff of 1 (=4-3).
- If you both bid 1, then you have a 50% chance of being the winner- in which case your payoff is 3 (from paying a price of 1)—and a 50% chance that you're not the winner- in which case your payoff is zero; the expected payoff is then $\frac{3}{2}$.

	1	2	3	4	5
1	$\frac{3}{2}$, 1	0,1	0,0	0,−1	0,−2
2	2,0	1, $\frac{1}{2}$	0,0	0,−1	0,−2
3	1,0	1,0	$\frac{1}{2}$, 0	0,−1	0,−2
4	0,0	0,0	0,0	0,$-\frac{1}{2}$	0,−2
5	−1,0	−1,0	−1,0	−1,0	$-\frac{1}{2}$,−1

Table: The strategic form of the first-price auction

First-price Auction (cont'd)

- Bidding 5 is strictly dominated by bidding 4. clearly you don't want to bid that much.
- You probably don't want to bid 4 since that is weakly dominated by any lower bid.

Your College

You	1	2	3	4	5
1	$\frac{3}{2}$, 1	0,1	0,0	0,−1	0,−2
2	2,0	1, $\frac{1}{2}$	0,0	0,−1	0,−2
3	1,0	1,0	$\frac{1}{2}$, 0	0,−1	0,−2
4	0,0	0,0	0,0	0,$-\frac{1}{2}$	0,−2
5	−1,0	−1,0	−1,0	−1,0	$-\frac{1}{2}$,−1

Table: The strategic form of the first-price auction

First-price Auction (cont'd)

- The minimum bid of 1 is also weakly dominated.
- We eliminat bids 1, 4 and 5 because they are either strictly or weakly dominated.
- Can we say more? Unfortunately, no

Your College

You	1	2	3	4	5
1	$\frac{3}{2}$, 1	0,1	0,0	0,−1	0,−2
2	2,0	1, $\frac{1}{2}$	0,0	0,−1	0,−2
3	1,0	1,0	$\frac{1}{2}$, 0	0,−1	0,−2
4	0,0	0,0	0,0	0,$-\frac{1}{2}$	0,−2
5	−1,0	−1,0	−1,0	−1,0	$-\frac{1}{2}$,−1

- Either a bid of 2 or 3 may be best, depending on what the other bidder submits.

Summary: Iterative Strict and Rationality

- Players maximize their payoffs.
  - They don't play strictly dominated strategies
  - They don't play strictly dominated strategies, given what remains...
- Nash equilibria are a subset of what remains
- Do we see such behavior in reality?

Feeding Behavior among Pigs and Iterated Strict Dominance

- Experiment by B.A. Baldwin and G.B. Meese (1979) "Social Behavior in Pigs Studied by Means of Operant Conditioning," Animal Behavior, Vol 27, pp 947-957. (See also J. Harrington (2011) Games, Strategies and Decision Making, Worth Publishers).
- Two pigs in cage, one is larger: "dominant" (sorry for the terminology...).
- Need to press a lever to get food to arive
- Food and lever are at opposite sides of cage
- Run to press and the other pig gets the food...

Feeding Behavior among Pigs and Iterated Strict Dominance

- 10 units of food- the typical split:
  - if large gets to food first, then 1,9 split (1 for small, 9 for large),
  - if small gets to food first then 4, 6 split,
  - if they get to food at the same time then 3, 7 split,
  - Pressing the lever costs 2 units of food in energy

Small/Large	Press	Wait
Press	1,5	$-1, 9$
Wait	4,4	0,0

Let us solve via iterative elimination of strictly dominated strategies

Small/Large	Press	Wait
Press	1,5	$-1, 9$
Wait	4,4	0,0

Pigs Behavior: Frequency of pushing the lever per 15 minutes, after ten tests (learning...) Baldwin and Meese (1979)

- Experiment was devised by Bladwin and Meese.
- It has two domestic pigs:
  - One is the dominate & the other is the subordinate.
  - Which pig will press the lever and run and which will be sitting by the food?

	Alone	Together
LargePigs	75	105
SmallPigs	70	5

Iterative Strict Dominance

- Are pigs rational? Do they know game theory?
- They do seem to learn and respond to incentives
- Learn not to play a strictly dominated strategy ...
- Learn not to play a strictly dominated strategies out of what remains...
- Learning, evolution, and survival of the fittest: powerful game theory tools

Maxmin Strategies

- Player i's **maxmin strategy** is a strategy that maximizes i's worst-case payoff, in the situation where all the other players (whom we denote $-i$) happen to play the strategies which cause the greatest harm to i.
- The **maxmin value** (or **safety level**) of the game for player i is that minimum payoff guaranteed by a maxmin strategy.

Definition (Maxmin)

The maxmin strategy for player i is $arg\ max_{s_i} min_{s_{-i}} u_i\ (s_i, s_{-i})$, and the maxmin value for player i is $max_{s_i} min_{s_{-i}} u_i\ (s_i, s_{-i})$

- Why would i want to play a maxmin strategy?

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- Why would i want to play a maxmin strategy?
  - a conservative agent maximizing worst-case payoff
  - paranoid agent who believes everyone is out to get him

Minmax Strategies

- Player i's **minmax strategy** against player $-i$ in a $2-$player game is a strategy that minimizes $-i$'s best-case payoff, and the **minmax value** for i against $-i$ is payoff.

Definition (Minmax, 2-player)

In a two-player game, the minmax strategy for player i against player $-i$ is $arg\ min_{s_i} max_{s_{-i}} u_{\boldsymbol{-i}}\left(s_i, s_{-i}\right)$, and player -i's minmax value is $min_{s_i} max_{s_{-i}} u_{\boldsymbol{-i}}\left(s_i, s_{-i}\right)$.

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Minmax Strategies

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- Why would i want to play a minmax strategy?
  - to punish the other agent as much as possible

Minmax Theorem

Theorem (Minmax, von Neumann, 1928)

In any finite, 2-player, zero-sum game, in any Nash equilibrium each player receives a payoff that is equal to both his maxmin value and his minmax value.

1. Each player's maxmin value is equal to his minmax value. The maxmin value for player 1 is called the **value of the game**.

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1. Each player's maxmin value is equal to his minmax value. The maxmin value for player 1 is called the **value of the game**.
2. For both players, the set of maxmin strategies coincides with the set of minmax strategies.

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1. Each player's maxmin value is equal to his minmax value. The maxmin value for player 1 is called the **value of the game**.
2. For both players, the set of maxmin strategies coincides with the set of minmax strategies.
3. Any maxmin strategy profile (or, equivalently, minmax strategy profile) is a Nash equilibrium. Furthermore, these are all the Nash equilibria. Consequently, all Nash equilibria have the same payoff vector (namely, those in which player 1 gets the value of the game).

Minmax Theorem (cont'd)

Theorem (Minmax, von Neumann, 1928)

In any finite, 2-player, zero-sum game, in any Nash equilibrium each player receives a payoff that is equal to both his maxmin value and his minmax value.

Proof:

We consider a game with two players.

- ✔ Player 1 make choice $k \in \{1, \dots, n\}$ & Palyer 2 make choice $l \in \{1, \dots, m\}$
- ✔ Player 1 then makes a payment of P_{kl} to Player 2 where $P \in R^{n\times m}$ is payoff matrix for game.
- ☞ **The goal of player 1 is to make the payment as small as possible, while the goal of player 2 is to maximize it.**
- ✔ The players use randomized or mixed strategies
  $$\mathbf{prob}(k = i) = u_i, \quad i = 1, \dots, n \quad \& \quad \mathbf{prob}(l = i) = v_i \quad i = 1, \dots, m$$
- ✔ The expected payoff from player 1 to player 2 is $\sum_{k=1}^{n} \sum_{l=1}^{m} u_k v_l P_{kl}$
- ☞ Player 1 wishes to choose u to minimize $u^T P v$, while player 2 wishes to choose v to maximize $u^T P v$.
- ✔ . . .
  $$\begin{array}{lll} \textit{minimize} & max_{i=1,\dots,m}(P^T u)_i = \textit{maximize} & min_{i=1,\dots,n}(P^T v)_i \\ \textit{s.t.} & u \succeq 0, \quad 1^T u = 1 \qquad\qquad \textit{s.t.} & v \succeq 0, \quad 1^T v = 1 \end{array}$$

2×2 Zero-sum Games

- Minmax or maxmin produces the same result as method for finding NE in general 2×2 games;
- Check against penalty kick game.

Penalty Kick Game

		Goalie	
		L	R
Kicker	L	$0.6, 0.4$	$0.8, 0.2$
	R	$0.9, 0.1$	$0.7, 0.3$

- How does the kicker maximize his minimum?

Penalty Kick Game

		Goalie	
		L	R
Kicker	L	0.6, 0.4	0.8, 0.2
	R	0.9, 0.1	0.7, 0.3

- How does the kicker maximize his minimum?

$$\max_{s_1}\min_{s_2}[s_1(L)s_2(L)\times 0.6+s_1(L)s_2(R)\times 0.8+s_1(R)s_2(L)\times 0.9+s_1(R)s_2(R)\times 0.7]$$

Penalty Kick Game (cont'd)

Goalie

Kicker		L	R
	L	0.6, 0.4	0.8, 0.2
	R	0.9, 0.1	0.7, 0.3

- What is his minimum?

Penalty Kick Game (cont'd)

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		L	R
Kicker	L	$0.6, 0.4$	$0.8, 0.2$
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- What is his minimum?

$$\min_{s_2}[s_1(L)s_2(L)\times 0.6+s_1(L)s_2(R)\times 0.8+s_1(R)s_2(L)\times 0.9+s_1(R)s_2(R)\times 0.7]$$

$$= \min_{s_2}\left[\begin{array}{c} s_1(L)s_2(L)\times 0.6 + s_1(L)(1-s_2(L))\times 0.8 + (1-s_1(L))s_2(L)\times \\ 0.9 + (1-s_1(L))(1-s_2(L))\times 0.7 \end{array}\right]$$

Penalty Kick Game (cont'd)

		Goalie	
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$$= \min_{s_2}\begin{bmatrix} s_1(L)s_2(L)\times 0.6 + s_1(L)(1-s_2(L))\times 0.8 + (1-s_1(L))s_2(L)\times \\ 0.9 + (1-s_1(L))(1-s_2(L))\times 0.7 \end{bmatrix}$$

$$= \min_{s_2}[(0.2 - s_1(L)\times 0.4)\times s_2(L) + (0.7 + s_1(L)\times 0.1)]$$

Penalty Kick Game (cont'd)

		Goalie	
		L	R
Kicker	L	$0.6, 0.4$	$0.8, 0.2$
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$$= \min_{s_2}[(0.2-s_1(L)\times 0.4)\times s_2(L)+(0.7+s_1(L)\times 0.1)]$$

$$\Rightarrow 0.2-s_1(L)\times 0.4=0$$

$$\Rightarrow s_1(L)=\frac{1}{2},\quad s_1(R)=\frac{1}{2}$$

Penalty Kick Game (cont'd)

		Goalie	
		L	R
Kicker	L	$0.6, 0.4$	$0.8, 0.2$
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- How does the goalie minimize the kicker's maximum?

Penalty Kick Game (cont'd)

Goalie

Kicker		L	R
	L	$0.6, 0.4$	$0.8, 0.2$
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- How does the goalie minimize the kicker's maximum?

$$\min_{s_2}\max_{s_1}[s_1(L)s_2(L)\times 0.6+s_1(L)s_2(R)\times 0.8+s_1(R)s_2(L)\times 0.9+s_1(R)s_2(R)\times 0.7]$$

Penalty Kick Game (cont'd)

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- What is the kicker's maximum?

Penalty Kick Game (cont'd)

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- What is the kicker's maximum?

$$\max_{s_1}[s_1(L)s_2(L) \times 0.6 + s_1(L)s_2(R) \times 0.8 + s_1(R)s_2(L) \times 0.9 + s_1(R)s_2(R) \times 0.7]$$

$$= \max_{s_1} \begin{bmatrix} s_1(L)s_2(L) \times 0.6 + s_1(L)(1 - s_2(L)) \times 0.8 + (1 - s_1(L))s_2(L) \times 0.9 \\ +(1 - s_1(L))(1 - s_2(L)) \times 0.7 \end{bmatrix}$$

Penalty Kick Game (cont'd)

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$$\max_{s_1}[s_1(L)s_2(L)\times 0.6 + s_1(L)s_2(R)\times 0.8 + s_1(R)s_2(L)\times 0.9 + s_1(R)s_2(R)\times 0.7]$$

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$$=\max_{s_1}[(0.1-s_2(L)\times 0.4)\times s_1(L)+(0.7+s_2(L)\times 0.2)]$$

$$\Rightarrow 0.1-s_2(L)\times 0.4=0$$

$$\Rightarrow s_2(L)=\tfrac{1}{4},\quad s_2(R)=\tfrac{3}{4}$$

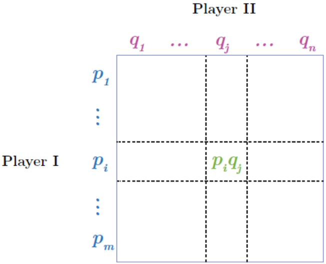
Computing Minmax

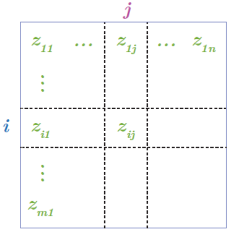
- For 2 players minmax is solvable with LP (Linear Programming).

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & u \succeq 0, \quad 1^T u = 1 \\ & P^T u \preceq t1 \end{array}$$

Correlated Equilibrium: Intuition

- **Correlated Equilibrium** (informal): a randomized assignment of (potentially correlated) action recommendations to agents, such that nobody wants to deviate.
- In a Nash equilibrium, the probability that player I plays i and player II plays j is the product of the two corresponding probabilities (in this case $p_i q_j$), whereas a correlated equilibrium puts a probability, say z_{ij} , on each pair (i, j) of strategies.





Correlated Equilibrium: Example

- Consider again **Battle of the Sexes**
  - In this game, there are two pure Nash equilibria (F, F), (B, B).
  - There is also a mixed Nash equilibrium yields each player an expected payoff of $\frac{2}{3}$.
  - How might this couple decide between the two pure Nash equilibria?
    - Intuitively, the best outcome seems a 50-50 (based on a flip of a single coin) split between (F, F), (B, B).
    - The expected payoff to each player in this so-called correlated equilibrium is $0.5 * 2 + 0.5 * 1 = 1.5$

	B	F
B	$2, 1$	$0, 0$
F	$0, 0$	$1, 2$

Correlated Equilibrium: Example (cont'd)

- What is the natural solution here?
  - A **traffic light**: a fair randomizing device that tells one of the agents to go and the other to wait.
- We could use the same idea to achieve the fair outcome in battle of the sexes.
- Benefits:
  - the negative payoff outcomes are completely avoided
  - fairness is achieved
  - the sum of social welfare can exceed that of any Nash equilibrium

	go	wait
go	$-10, -10$	$1, 0$
wait	$0, 1$	$-1, -1$